JOINT CUMULANT AND CORRELATION BASED SIGNAL SEPARATION WITH APPLICATION TO EEG DATA ANALYSIS

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ABSTRACT

Current methods in Blind Source Separation (BSS) utilize either the higher order statistics or the time delayed cross-correlations to perform signal separation. In this paper we investigate a method for source separation which utilizes joint information from higher order statistics and delayed cross-correlations. The algorithm is motivated by problems in analysis of Electroencephalography (EEG) data. We use an EEG analysis example to demonstrate that the Joint Cumulant and Correlation based (JCC) algorithm obtains better source separation than either of the group methods based on higher order statistics or time delayed cross correlations.

1. INTRODUCTION

Most methods in Blind Source Separation (BSS) and Independent Components Analysis (ICA) use higher order statistics and spatial diversity to force the statistical independence of the solution components. An alternative approach as the so called Second Order Blind Identification (SOBI) [1] exploits the spatial and spectral diversities of the sources. This is achieved by utilizing time delayed cross-correlations to impose at different time delays a decorrelation structure on the solution. These two approaches access different types of information and their relative performance depends entirely on the source properties. Accordingly, the first approach can separate at least one Gaussian source and performs poorly when the other sources are close to the Gaussian distribution. And the second approach can not separate sources with identical spectra shape and performs poorly when the spectra are close to each other. To overcome these problems, one can use a joint approach that utilizes together higher order statistics and time-delayed cross-correlations.

In problems such as EEG source analysis, the choice of a BSS algorithm is particularly difficult because it is largely impossible to assess the quality of separation or even that the estimated sources have any relation to the actual centers of activity in the brain. Several higher order algorithms have been used in EEG data analysis [3, 4, 2]. Yet, other groups [5, 6] found that delayed cross-correlation based algorithms, namely SOBI, obtains more plausible separation in EEG and MEG data.

Here, we introduce a method that uses joint cumulant and correlation matrices. Termed JCC, the method exploits both higher order statistics and time-delayed cross correlations by performing joint diagonalization of a set of cumulant matrices, namely the eigen matrices of the data quadricovariance, and a set of time-delayed cross-correlation matrices.

As a result, we are able to separate sources that are not separable by either the higher order statistics or the time delayed cross-correlation based BSS algorithms alone. The performance of the JCC algorithm is demonstrated using a simulated set of sources and an example from EEG analysis. The EEG example was designed to be able to ascertain the quality of separation. The EEG data in this example contains brain and ocular sources, where the eye movement is primarily horizontal and is decoupled from the vertical eye blink. The quality of separation of ocular sources can be judged on how well the methods can separate saccades and the blink from the rest of the activity.

2. BACKGROUND

2.1. Data model

We consider the n-dimensional vector of sensor signals defined as $\mathbf{x}(t) = [x_1(t), \cdots, x_n(t)]^T$, generated by an unknown linear model:

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) \tag{1}$$

where $\mathbf{s}(t) = [s_1(t), \cdots, s_m(t)]^T$ is the m-dimensional vector whose elements are called sources. The matrix $\mathbf{A}$ is

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called a mixing matrix. The source signals $s_j(t),\ j = 1, \cdots, m$ ($m \leq n$), are assumed independent.

It is well known that due to the lack of prior information, the problem of source separation has two inherent ambiguities:

Permutation ambiguity: It is not possible to know the original labeling of the sources; hence, any permutation of the estimated sources is also a satisfactory solution.

Scaling ambiguity: It is inherently impossible to uniquely identify the source signals because the exchange of a fixed scalar factor between a source signal and the corresponding column of the mixture matrix $\mathbf{A}$ does not effect the observations.

2.2. Algebraic identification approaches

Herein, we outline two blind identification approaches based on two step processing; the first step is common to the two approaches and consists of whitening the data by a *sphering matrix* $\mathbf{W}$ in order to transform the mixing matrix $\mathbf{A}$ into a unitary matrix $\mathbf{U}$ [1]. The second step consists then of retrieving this unitary matrix by joint diagonalizing a set of data correlation matrices when considering the first approach referred to as Second Order Blind Identification (SOBI) and a set of eigen matrices of the data quadricovariance referred to as Joint Approximate Decomposition of Eigen matrices (JADE) . Note that the first step is nothing than the Principal Component Analysis (PCA).

2.2.1. SOBI algorithm

Under the linear data model of equation (1) and hypothesis of sources with different spectra shape, the time delayed cross-correlation matrices take the following simple structure:

$$\begin{aligned} \mathbf{R}(\tau) &= E[\mathbf{x}(t)\mathbf{x}(t-\tau)^T] \\ &= \mathbf{A}\mathbf{R}_s(\tau)\mathbf{A}^H, \quad \text{for } \tau \neq 0 \end{aligned} \tag{2}$$

where $\mathbf{R}_s = \text{diag}[\rho_1, \cdots, \rho_m]$ is time delayed cross correlation matrix of the sources whose entries $\rho_i = E[s_i(t)s_i(t-\tau)^T],\ i = 1, \cdots, m$ are the correlation coefficients of the sources.

The whitened correlation matrices $\underline{\mathbf{R}}(\tau)$ are than given by

$$\forall \tau \neq 0 \quad \underline{\mathbf{R}}(\tau) = \mathbf{W}\mathbf{R}(\tau)\mathbf{W}^T = \mathbf{U}\mathbf{R}_s(\tau)\mathbf{U}^T \tag{3}$$

Since $\mathbf{U}$ is unitary and $\mathbf{R}_s(\tau)$ is diagonal, the latter just means that any whitened correlation matrix is diagonalized by the unitary transform $\mathbf{U}$. Hence, the missing unitary matrix $\mathbf{U}$ can be obtained by a joint diagonalization [1] of a set $\{\underline{\mathbf{R}}(\tau_i) | i = 1, \cdots, p\}$ of p whitened correlation matrices. The matrix $\mathbf{U}$ is 'unique' (i.e. up to permutation and phase shifts) if and only if for any pair (i, j) of sources, there exists at least one lag τ_l in $(\tau_1, \cdots, \tau_p)$ such that $\rho_i(\tau_l) \neq \rho_j(\tau_l)$. We omit the proof of this statement that can be found in [1]. Once the unitary matrix $\mathbf{U}$ is obtained, the mixing matrix is estimated by $\hat{\mathbf{A}} = \mathbf{W}\mathbf{U}$ and the unmixing matrix is then given by $\mathbf{U}^T\mathbf{W}^\#$, where $^\#$ denotes the pseudo-inverse.

2.2.2. JADE algorithm

To a n-dimensional random vector $\mathbf{x}$ with 4th-order cumulants, a quadricovariance $\mathbf{Q}$ is associated. It is defined as the linear matrix-to-matrix mapping: $M \rightarrow N = \mathbf{Q}(M)$ where M and N are $n \times n$ matrices related by

$$[N]_{ij} = \sum_{kl} Cum(x_i, x_j, x_k, x_l)[M]_{kl} \tag{4}$$

where $Cum(x_i, x_j, x_k, x_l)$ denotes the 4th-order cumulants of $\mathbf{x}$ [7]. It is shown that since the set of the $n \times n$ matrices is a n^2-dimensional linear space, there exist n^2 real numbers λ_r and n^2 orthonormal matrices M_r, verifiying $\mathbf{Q}(M_r) = \lambda_r M_r$, $r = 1, \cdots, n^2$. Note that $\mathbf{Q}$ is actually a 4th-order tensor and the M_r matrices are the eigen-matrices of $\mathbf{Q}$ associated to its eigen values λ_r. It is proved [7] that the quadricovariance $\mathbf{Q}$ has exactly rank n so that only n out of n^2 eigenvalues are non zero.

Under the linear data model of equation (1) and hypothesis of independent sources, the eigen matrices of the quadricovariance of the whitened data have the following structure,

$$\underline{\mathbf{M}}_r = \mathbf{U}\mathbf{D}_r\mathbf{U}^T, \quad \mathbf{D}_r = Diag[\cdots, \mathbf{u}_p^T \underline{\mathbf{M}}_r \mathbf{u}_p, \cdots] \tag{5}$$

where $r = 1, \cdots, n$ and $\mathbf{u}_p$ denotes the p-th column of matrix $\mathbf{U}$. Note that the n non zero eigen values associated to the eigen matrices $\underline{\mathbf{M}}_r$ correspond to the kurtosis of the sources.

According to (5), the missing unitary matrix $\mathbf{U}$ can be obtained by a joint diagonalization of the n eigen matrices $\underline{\mathbf{M}}_r$ [7]. Once the unitary matrix $\mathbf{U}$ is obtained, the mixing matrix is estimated by $\hat{\mathbf{A}} = \mathbf{W}\mathbf{U}$ and the unmixing matrix is then given by $\mathbf{U}^T\mathbf{W}^\#$.

3. THE JCC ALGORITHM

SOBI and JADE both start by sphering the data and then perform the joint diagonalization of a set of matrices. SOBI uses time delayed cross-correlation matrices and JADE uses quadricovariance eigen matrices. These two groups of matrices contain different information, namely the correlation information and the cummulant information. To take advantage of both information, the Joint Cumulant and Correlation algorithm suggests to simultaneously diagonalize a set of the two groups of matrices.

Under the linear data model of equation (1) , the eigen matrices of the quadricovariance of the whitened data and the whitened data correlation matrices have the following structure,

$$\underline{\mathbf{M}}_r = \mathbf{U}\mathbf{D}_r\mathbf{U}^T, \tag{6}$$
$$\underline{\mathbf{R}}(\tau) = \mathbf{U}\mathbf{R}_s(\tau)\mathbf{U}^H, \quad \text{for } \tau \neq 0 \tag{7}$$

where $\mathbf{U}$ is unitary and $\mathbf{D}_r$ and $\mathbf{R}_s(\tau)$ are both diagonal. Hence, the unitary matrix $\mathbf{U}$ can be obtained by a joint diagonalization of a set of the matrices $\underline{\mathbf{M}}_r$ and $\underline{\mathbf{R}}(\tau)$. The program is completed by estimating the mixing matrix by $\mathbf{WU}$ and the unmixing matrix by $\mathbf{U}^T\mathbf{W}^{\#}$.

One may expect that JCC will perform better than SOBI and JADE. This is not always true because the incorporation in the joint diagonalization procedure, of non accurate estimates of either the cumulant or correlation matrices will degrade the performance. However, JCC will be of great importance in examples where the identification conditions of both SOBI and JADE are not meet. Hence in this case, JCC will be able to succeed where both SOBI and JADE will fail to separate all the sources.

4. SIMULATION

Herein, we consider 2 Gaussian distributed sources with different spectrums and 2 white binary sources. The original sources are displayed in Figure 1. Figure 2 shows the spectrums and the histograms (sample distributions) of the 4 sources. Note that the identification conditions of both SOBI and JADE are not meet in this example. Four linear mixtures of the sources are generated by a random mixing matrix. The mixed signals are shown in Figure 3. The estimated sources by SOBI, JADE and JCC are plotted in Figures 4, 5 and 4, respectively. Figure 4 shows that SOBI has succeeded to separate only the two sources that have different spectrums. Figure 5 shows that JADE has succeeded to separate only the binary sources. Figure 6 shows that JCC algorithm has succeeded to separate all the sources. The same conclusion is achieved when we compute the matrix,

$$\mathbf{P} = \hat{\mathbf{A}}^{-1}\mathbf{A} \tag{8}$$

where $\mathbf{A}$ and $\hat{\mathbf{A}}$ are the true and estimated mixing matrix, respectively. Because of the ambiguities stated in Section 2.1, this matrix is close to a permutation matrix for a good separation. Accordingly, we have obtained for the three algorithms the following matrices:

$$\mathbf{P}_{sobi} = \begin{bmatrix} \mathbf{1.00} & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & \mathbf{0.42} & \mathbf{0.58} \\ 0.00 & 0.00 & \mathbf{0.57} & \mathbf{0.42} \\ 0.00 & \mathbf{1.00} & 0.00 & 0.00 \end{bmatrix} \tag{9}$$

$$\mathbf{P}_{jade} = \begin{bmatrix} \mathbf{0.46} & \mathbf{0.52} & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & \mathbf{1.00} \\ \mathbf{0.55} & \mathbf{0.49} & 0.00 & 0.00 \\ 0.00 & 0.00 & \mathbf{1.00} & 0.00 \end{bmatrix} \tag{10}$$

$$\mathbf{P}_{jcc} = \begin{bmatrix} 0.00 & 0.00 & 0.00 & \mathbf{1.00} \\ 0.00 & \mathbf{1.00} & 0.00 & 0.00 \\ 0.00 & 0.00 & \mathbf{1.06} & 0.00 \\ \mathbf{0.95} & 0.00 & 0.00 & 0.00 \end{bmatrix} \tag{11}$$

The 3-rd and 4-th columns of the matrix $\mathbf{P}_{sobi}$ related to SOBI show that the two corresponding sources are still mixed at the output of the separator. The 1-st and 2-nd columns of the matrix $\mathbf{P}_{jade}$ related to JADE show that the two corresponding sources are still mixed at the output of the separator. In contract to $\mathbf{P}_{sobi}$ and $\mathbf{P}_{jade}$, the matrix $\mathbf{P}_{jcc}$ that is related to the JCC algorithm is a quasi permutation matrix, which shows that all the sources have been perfectly separated.

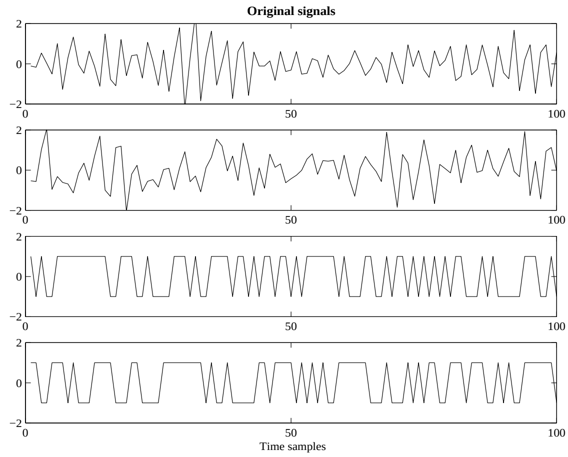

Fig. 1. Original sources.

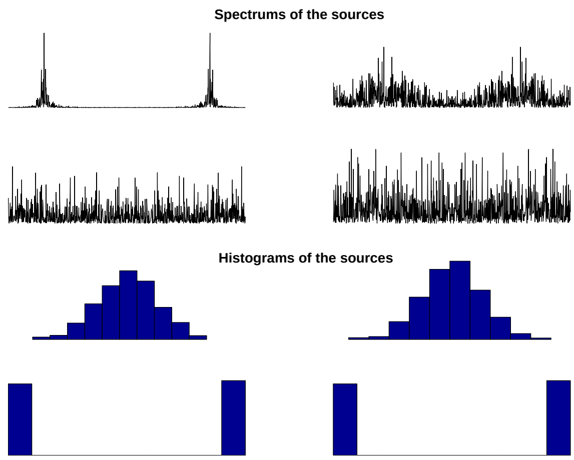

Fig. 2. Spectrums and histogram of the sources.

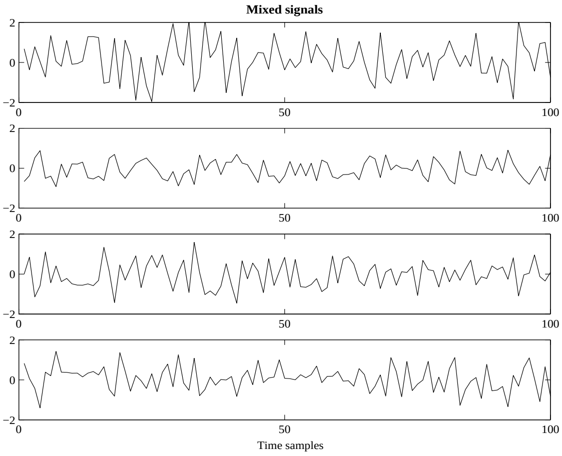

Fig. 3. Mixed sources.

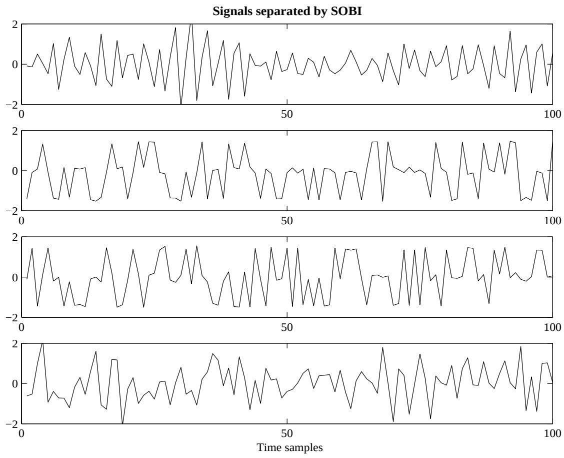

Fig. 4. Separated sources by SOBI.

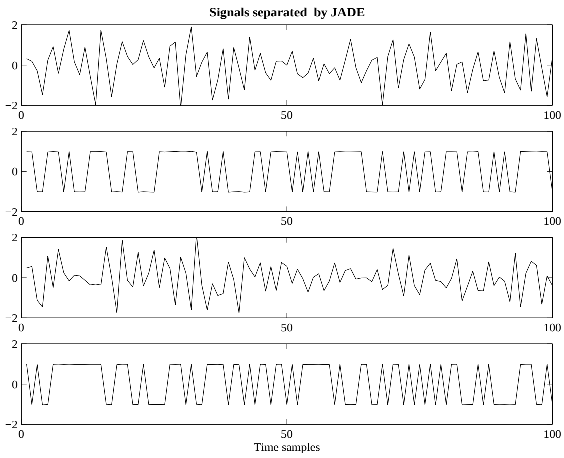

Fig. 5. Separated sources by JADE.

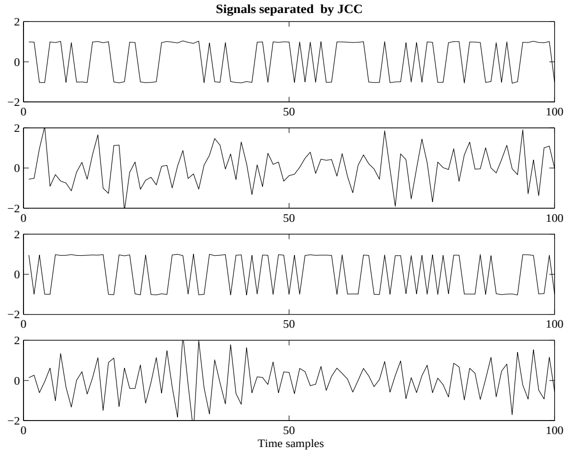

Fig. 6. Separated sources by JCC.

5. EEG DATA ANALYSIS

A difficulty in using source separation in EEG analysis is that no objective way exists to evaluate the accuracy of the obtained source separation. One goal of this study is to design an EEG experiment to validate the performance of various source separation algorithms. Since the eyes constitute strong dipolar bioelectric sources whose signals mix with the signals coming from the brain, we set up an experiment where the subjects move their eyes in a predetermined pattern and we use these known ocular sources to assess how well they are separated by a given algorithm. This experiment offers an objective way of evaluating the quality of separation of a subset of EEG sources.

In the experiment described, the subjects were asked to read four lines of text presented on the computer screen. Thus the eye movements were primarily horizontal with quick vertical saccades between the lines. The signals generated by eye blinks are generated primarily by the vertical movement of the eye lid, although a small vertical movement of the eye is also present. Eye blinks produce very strong electric signals that add to the mixture of brain and ocular signals recorded in EEG. Because the signal is coming primarily from the eye lid, it is decoupled from the ocular signal, but the two electric sources, the eye lid and the eyeball, are located very close to each other. It is thus difficult for source separation algorithms to separate out these closely spaced sources. The eyes' horizontal movement and the blink generated by a vertical eye lid movement provide signals that let us evaluate how cleanly the two motion components, and thus the sources, are recovered by the source separation algorithms.

The data were recorded using a 32 channel EEG recording system. Figure 7 shows 10 channels of the recorded data that were used in the analysis. Channels 1, 6, 7, 8, and 9 are ocular channels, those recording the electrodes were placed around the two eyes. A blink occurred at the end of the experiment. The blink is reflected in practically all the channels, except channel 9 which measured the generated electric field components that were purely due to the horizontal motions of dipoles. Thus this channel provided a very close measure of the actual ocular sources. It is the only channel that did not reflect the vertical eye blink. Channels 7 and 8 reflect the signals due to the vertical motion of the eyes, the small vertical saccades where the transitions between lines of the text was made, and the motion of the eye lid during the blink. Channel 6 reflects a mixture of electric signals generated from both horizontal and vertical movements of the eyes and eye lids. Channels 2, 3, 4 and 5 were located toward the front of the head and record the signal due to the eye blink and brain activity. We found that 10 channels (or even less) are sufficient to separate eye components from the data.

Figure 8 shows separation from 10 channels of the EEG data using SOBI. Three components with ocular characteristics are observed in the separation (Sources 1, 8, and 10 in Figure 8). Sources 1 and 8 in Figure 8 represent the two eyes. The eye blink, that is the eye lid source, however, is not separated well from the ocular sources and the brain signals. The eye blink component is clearly present in components 6, 7, 8, and 10. Both component 8 and 10 also show saccade movements in them. This poor ability to separate out a sharp delta function-like blink is consistent with the expected performance of SOBI [1]. The time delayed cross-correlations are near zero for such sources, which causes poor separation of such signals by the SOBI algorithm. Thus SOBI separates out the two individual eye sources, but has poor performance in separating eye blinks.

Figure 9 shows separation from the same 10 channels of the EEG data using JADE. An ocular component (Source number 3 in Figure 9) and a vertical motion eye lid component (Source number 10 in Figure 9) are identified. The eye blink can be seen only in component 10 and thus is separated cleanly from the rest of the source. JADE, however, does not identify two separate ocular sources. The components it identified are noisier than those obtained by SOBI and it does not identify the fourth eye saccade, which occurs right before the blink. JADE performance also degraded noticeably when run using all 32-channels (not shown here).

Figure 10 shows separation by the JCC algorithm from the 10 channels of the EEG data shown in Figure 7. The JCC separation clearly identifies the two ocular sources (Sources 1 and 4 in Figure 10), and the eye blink component (Source 8 in Figure 10), which is cleanly separated from the rest of the sources. The ocular and eye lid components contain minimal noise as in the SOBI separation. Thus JCC separation in this example combines properties of the SOBI and the JADE algorithms to identify both the ocular and the eye blink sources, which the latter two algorithms were unable to do individually. The rest of the JCC separated components, which we assume are due to the brain activity, look different from the corresponding components extracted by either SOBI or JADE. This demonstrates how the quality of the separation of the ocular and eye lid sources affects the separation of the rest of the sources.

6. DISCUSSION

This study is motivated by the need for a rigorous evaluation of the accuracy of separation of EEG sources provided by the different source separation algorithms. We have designed an experiment in which we are able to evaluate such performance for at least some of the sources represented in the EEG data. We evaluated three groups of blind source separation algorithms, one based on higher order statistics, another based on time delayed cross-correlations, and a joint method that utilizes both of these criteria. In our experiments, the method based on the joint separation criteria was able to separate the ocular and eye lid sources completely, while the methods based on just one of the individual criteria were not able to recover these sources without errors. The study presented indicates that methods based on joint higher order and time delayed cross-correlation criteria may be better suited for separation of at least the ocular and eye related sources in EEG data. How well the different methods can separate EEG sources due to cortical activity needs to be investigated further.

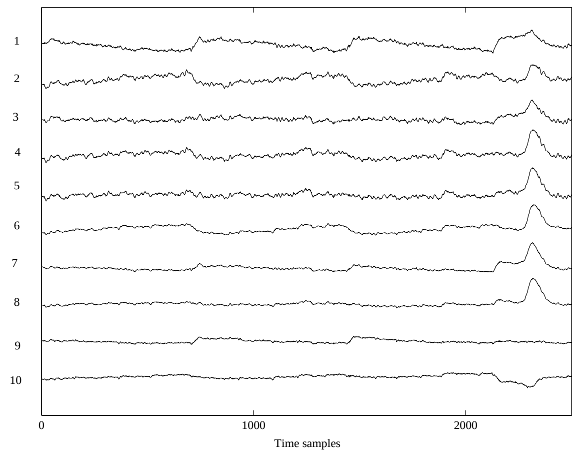

Fig. 7. 10 channels of the recorded EEG data.

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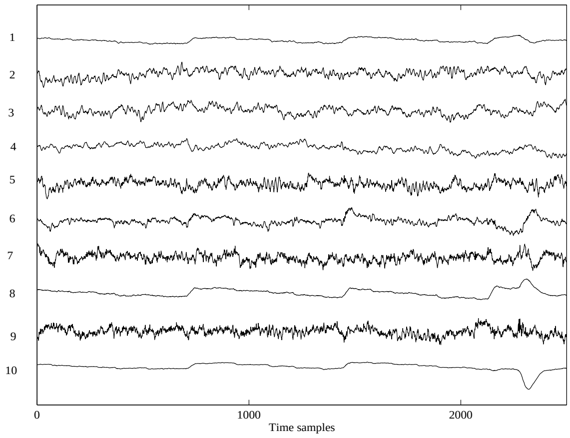

Fig. 8. EEG data separated by SOBI.

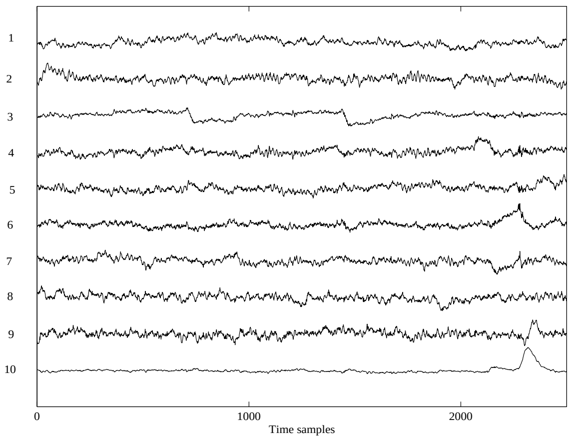

Fig. 9. EEG data separated by Jade.

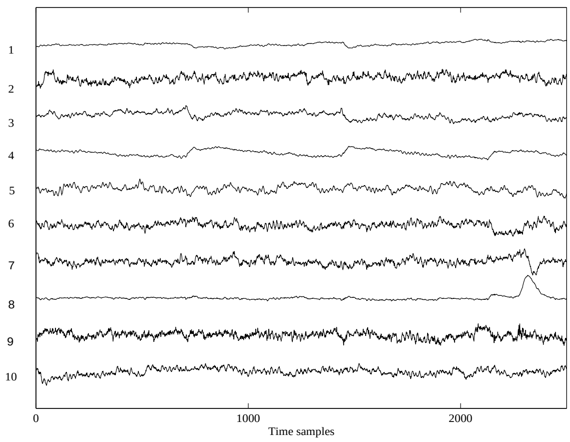

Fig. 10. EEG data separated by JCC.

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