

HUYGENS-FRESNEL PRINCIPLE GENERATES NONLINEAR ICA REDUCIBLE TO SOLVABLE INCOHERENT LIMIT LINEAR ICA

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Abstract

Reticle trackers have been used successfully with a beam splitter for tracking and discrimination of several moving incoherent (heat) optical sources in the mathematical framework called Independent Component Analyses (ICA). Here we further explore the theoretical basis of the coherent and partially coherent illumination by laser for the possibility of blind source de-mixing. An application of the partial coherence theory and the Huygens-Fresnel principle is utilized to formulate the problem. When incoherence is assumed a linear ICA model is obtained while in most general case of either partially or totally coherent optical radiation the resulting signal model is inherently nonlinear. It can be transformed into linear one under very special condition that assumes no relative motion between the radiating sources. In the most general case of partially coherent radiation, tracking of the several moving optical sources by using the beam splitter based reticle trackers is possible either by using ICA algorithms developed for undercomplete representation or by introduction of one additional sensor.

1.0 Introduction

Reticle trackers are considered to be the classical approach for estimating the position of a target in a considered field of view and are widely used in IR seekers, [3-13]. Their advantage is simplicity and low cost, [10,11]. However, the major drawback of the reticle trackers has been proven to be sensitivity on the man-made clutters such as flares and jammers, [8,10]. Such limitation of the reticle systems in real world applications was very often due to the use of the single detector element, [10]. Several attempts to neutralize such problem are based on the introduction of the segmented focal plane arrays (FPA) behind the reticle. Since the advantage of the reticle seekers is simplicity and low cost the segmented FPA must be comprised of a small number of detectors so as not to become as complex and expensive as an imaging system with a full strength FPA. The problem still exists when the two sources are in space region acquired by the same detector element. Appropriate space resolution should be ensured requiring more detector elements. A new approach was proposed in [3,7] was extended in [4,5] and will be completed in this paper. It is based on the ICA theory and an appropriate modification of the optical tracker design. We present in Section 2 a brief description of the optical modulation theory while more details can be found in [8-12]. In Section 3 a rigorous derivation of the signal model of the modified optical tracker output signals is given. With the advent of Laser Radar, LIDAR, we explore the general case of either coherent and partially coherent laser sources or the incoherent heat sources for the possibility

of moving sources blind demixing and discrimination. An application of the statistical optics principles, partial coherence theory and Huygens-Fresnel principle, [1,2], is utilized to formulate the problem. Although, linear ICA is obtained a special solution for the incoherent sources this case is of practical interest. Therefore, we address in Section 4 the problem of the characterization of the linear part of the signal model derived in the Section 3. By using blind identification approach we show that convolutive mixing model could have non-minimum phase. We propose an adaptive frequency domain algorithm for separating experimental data obtained from the modified optical tracking device, [23]. In Section 5 experimental results are presented for the incoherent heat sources while conclusions are given in Section 6.

2.0 Optical Modulation Theory

The reticle system provides directional information for tracking and also suppresses unwanted background signals, [8,9], by performing modulation of the incident light flux. According to the type of the reticle and the relative motion produced by the scan pattern, the encoding method of the reticle may be classified into AM, FM and pulse code modulation. In addition, according to how the relative motion between the reticle and the optical spot is obtained we may classify reticle systems into fixed or moving reticle. When reticle is fixed relative motion can be obtained by using rotating mirror which causes the light beam and hence the spot to either nutate or rotate in relation to the fixed reticle. In the opposite case spot forming optics is fixed while reticle performs either nutation or spinning. The general case of one moving reticle system is illustrated with Fig. 1. Moving reticle is placed in the focal plane of the collecting optics, while field optics collects modulated light and focuses it on detector. The selective amplifier center frequency is usually the number of spoke pairs times the nutation or spinning frequency. The rising-sun reticle that is very often used in the nutating FM reticle trackers, [3,8,9], is shown on Fig. 2.

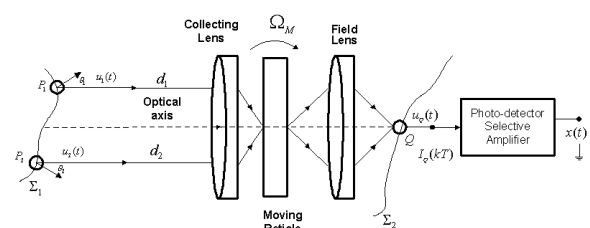


Figure 1. Moving reticle based optical tracker

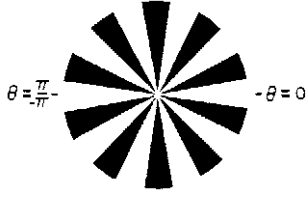


Figure 2. The rising-sun reticle

It can be noted from the previous discussion that relative motion between the spot and the reticle can be ensured either by nutation, [3,8,9], or rotation (spinning), [12]. In any case detector output voltage is proportional to the light irradiance behind the reticle according to [12,13]:

$$I(t) = I_p \int_0^R \int_{-\pi}^{\pi} s(r, \varphi) \delta[r - r_0, \theta - (\Omega t - \varphi_0)] r d\varphi dr \quad (1)$$

where $s(r, \varphi)$ is the reticle transmission function (rtf) and r and φ are spatial variables of the rtf ranging from 0 to R and $-\pi$ to π , respectively. Also let the reticle nutation or spinning rate be Ω in rads^{-1} and let r_0 and θ_0 be the spatial coordinates of a point source that is imaged onto the reticle. I_p in (1) is the peak irradiance of the point source through the rtf. Since the convolution of any function with delta function is the function located at the delta function coordinates the Eq. (1) becomes:

$$I(t) = I_p s(r_0, \Omega t - \varphi_0) \quad (2)$$

We shall derive Eq. (2) in Section 3 as a limiting case of the multi-source scenario by using more general approach based on the statistical optics principles, partial coherence theory and Huygens-Fresnel principle of the propagation of electromagnetic waves. In optical trackers that generate FM signal by means of the rising-sun reticle, Fig. 2, and nutation the rtf are of the form [9,11]:

$$s(r, \varphi, t) = I_p \cos[m\Omega t - m(r/a)\sin(\varphi)] \quad (3)$$

The optical spot performs circular motion, with radius a , around the center with coordinates (r, φ) relative to the center of the reticle. Necessary condition for Eq. (3) to hold is $(r/a)^2 \ll 1$. m in Eq. (3) is the number of spoke pairs of the reticle. Eq. (3) represents canonical form of the FM signal where frequency deviation from the carrier frequency is directly proportional with the spot r coordinate. So by using nutating rising-sun reticle, both directional information, distance and azimuth, are encoded in the reticle transmission function. Instead of using nutation the relative motion between the spot and the reticle can be obtained by simple rotation or spinning. More details about such systems can be found in [5,11,13].

3.0 Derivation of the Signal Model

We shall assume scenario shown on Fig. 1. Intensity at point Q (detector) is obtained as:

$$I_Q = \langle u(Q, t) u^*(Q, t) \rangle \quad (4)$$

where:

$$u(Q, t) = u(Q_1, t) + u(Q_2, t) \quad (5)$$

and $u(Q_1, t)$ is disturbance at point Q due to the point P_1 in the plane Σ_1 and $u(Q_2, t)$ is disturbance at point Q due to the point P_2 in plane Σ_2 . Those quantities can be obtained as functions of radiation at points P_1 and P_2 by application of the Huygens-Fresnel principle to the propagation of optical waves. Relation will be derived for quantity $u(Q_1, t)$ while for $u(Q_2, t)$ applies the full analogy. We will give derivation for the quasi-monochromatic or narrow-

band light since it is of practical interest. The purely monochromatic case is obtained as a special case of the quasi-monochromatic derivation. If the light is quasi-monochromatic then [1,2]:

$$u(Q_1, t) = \iint_{\Sigma_1} \frac{u(P_1, t - d_1/c)}{d_1} K(P_1, Q_1) \Lambda_1 dP_1 \quad (6)$$

where Λ_1 is the inclination factor that for the small diffraction angles θ_1 and θ_2 can be approximated with

$$\Lambda_1 = \frac{1}{j\lambda} \quad \text{where } \bar{\lambda} \text{ is central wavelength of the source}$$

emitting band. Because optical system is present between Σ_1 and Σ_2 its influence is taken into account introducing $K(P_1, Q_1)$ into integral (6). We shall assume here the ideal lenses and only rtf to be important so that:

$$K(P_1, Q_1) \cong s(r_1, \Omega t - \varphi_1) \quad (7)$$

Now intensity I_Q in Eq. (4) is obtained as:

$$I_Q = \langle u(Q_1, t) u^*(Q_1, t) \rangle + \langle u(Q_2, t) u^*(Q_2, t) \rangle + \langle u(Q_1, t) u^*(Q_2, t) \rangle + \langle u^*(Q_1, t) u(Q_2, t) \rangle \quad (8)$$

The first two parts in Eq. (8) represent intensities produced by the optical sources placed at points P_1 and P_2 respectively. Then for the quasi-monochromatic light it applies:

$$I(Q_1, t) = \langle u(Q_1, t) u^*(Q_1, t) \rangle = \frac{1}{\lambda^2} \iint_{\Sigma_1} \iint_{\Sigma_1} \frac{I(P_1, t)}{d_1^2} dP_1 dP_1 \times s(r_1, \varphi_1, t) \quad (9)$$

If we assume a point source at P_1 then:

$$u(P_1, t - \frac{d_1}{c}) = u(P_1, t - \frac{d_1}{c}) \delta(|P - P_1|) \quad (10)$$

Then Eq. (9) is reduced to:

$$I(Q_1, t) = \frac{1}{\lambda^2} \frac{I(P_1, t)}{d_1^2} s(r_1, \varphi_1, t) \quad (11)$$

In Eq. (15) and (17) as well as in analogous subsequent derivations it will be assumed that [4,5]:

$$s^2(r_1, \varphi_1, t) = s(r_1, \varphi_1, t) \\ \langle s(r_1, \varphi_1, t) \rangle = s(r_1, \varphi_1, t)$$

Derivation of the third and fourth part in Eq. (8) is becoming especially interesting. For a narrow-band light it applies the following:

$$\langle u(Q_1, t) u^*(Q_2, t) \rangle = \frac{1}{\lambda^2} \iint_{\Sigma_1} \iint_{\Sigma_1} \Gamma(P_1, P_2, t - \frac{d_2 - d_1}{c}) dP_1 dP_2 \times s(r_1, \varphi_1, t) s(r_2, \varphi_2, t) \quad (12)$$

For $\langle u^*(Q_1, t) u(Q_2, t) \rangle$ the same expression is obtained so it will not be derived. According to Ref. 2. (pp. 180 and 197, Eq. (5.2-31) and (5.4-7)) it applies for the quasi-monochromatic light:

$$\Gamma\left(P_1, P_2; \frac{d_2 - d_1}{c}\right) = J(P_1, P_2) \exp\left[-j \frac{2\pi}{\lambda} (d_2 - d_1)\right]$$

and Eq. (12) becomes:

$$\begin{aligned} \langle u(Q_1, t) u^*(Q_2, t) \rangle = & \\ \frac{1}{\lambda^2 d_1 d_2} \iint_{\Sigma_1} \iint_{\Sigma_1} J(P_1, P_2) \exp \left[-j \frac{2\pi}{\lambda} (d_2 - d_1) \right] dP_1 dP_2 & \\ \times s(r_1, \varphi_1, t) s(r_2, \varphi_2, t) & \end{aligned} \quad (13)$$

where $\Gamma(P_1, P_2, \tau)$ is mutual coherence and $J(P_1, P_2)$ is the mutual intensity of light at P_1 and P_2 . According to Ref. 2 (pp. 181 Eq. (5.2-30)-(5.2-33)) and Ref. 1 (1, pp. 507, Eq. (9) and (10)) mutual intensity can be expressed as:

$$J(P_1, P_2) \exp \left[-j \frac{2\pi}{\lambda} (d_2 - d_1) \right] = \sqrt{I(P_1) I(P_2)} \gamma_{12}(0)$$

where $\gamma_{12}(0)$ is mutual degree of coherence of the two sources u_1 and u_2 . Then Eq. (13) is transformed into:

$$\begin{aligned} \langle u(Q_1, t) u^*(Q_2, t) \rangle = & \\ \frac{1}{\lambda^2 d_1 d_2} \iint_{\Sigma_1} \iint_{\Sigma_1} \sqrt{I(P_1) I(P_2)} \gamma_{12}(0) dP_1 dP_2 & \quad (14) \\ \times s(r_1, \varphi_1, t) s(r_2, \varphi_2, t) & \end{aligned}$$

Assuming point sources at P_1 and P_2 we finally obtain:

$$\begin{aligned} \langle u(Q_1, t) u^*(Q_2, t) \rangle = \frac{1}{\lambda^2 d_1 d_2} \sqrt{I(P_1) I(P_2)} \gamma_{12}(0) & \\ \times s(r_1, \varphi_1, t) s(r_2, \varphi_2, t) & \end{aligned} \quad (15)$$

From Ref. 1 (pp. 507-508, Eq. (9)-(13)) and Ref. 2 (pp. 181 and 205, Eq. (5.2-37) and (5.5-14)-(5.5-16)) it applies for the quasi-monochromatic source:

$$\gamma_{12}(0) = |\gamma_{12}(0)| \cos(\beta_{12}) \quad (16)$$

where:

$$\beta_{12} = \arg \gamma_{12}(0) = \Phi(P_2) - \Phi(P_1) \quad (17)$$

and for the moving quasi-monochromatic sources it can be written:

$$\gamma_{12}(t) = |\gamma_{12}(t)| \cos \left[2\pi \frac{v}{\lambda} t + \Delta\Phi \right] \quad (18)$$

where v is relative velocity between the two points and $\Delta\Phi$ is some initial phase difference. Now Eq. (14) can be written as:

$$\begin{aligned} \langle u(Q_1, t) u^*(Q_2, t) \rangle = \frac{1}{\lambda^2 d_1 d_2} \sqrt{I(P_1) I(P_2)} & \\ \times \gamma_{12}(t) s(r_1, \varphi_1, t) s(r_2, \varphi_2, t) & \end{aligned} \quad (19)$$

where $\gamma_{12}(t)$ is given with Eq.(18) for quasi-monochromatic radiation and is in principle unknown function of time for the polychromatic radiation. Eq. (19) can be applied on the pure monochromatic sources replacing $\bar{\lambda}$ with λ_0 . The photo-current is obtained when the intensity I_Q , Eq. (8) and related Eq. (11) and (19), is expressed in terms of spectral irradiance and when detector spectral responsivity is taken into consideration giving for the quasi-monochromatic and purely monochromatic sources:

$$\begin{aligned} i(t) = \frac{A}{\lambda^2} \frac{I(P_1, \bar{\lambda}, t)}{d_1^2} R(\bar{\lambda}) \times s(r_1, \varphi_1, t) & \\ + \frac{A}{\lambda^2} \frac{I(P_2, \bar{\lambda}, t)}{d_2^2} R(\bar{\lambda}) \times s(r_2, \varphi_2, t) & \quad (20) \\ + \frac{1}{\lambda^2 d_1 d_2} \sqrt{I(P_1, \bar{\lambda}, t) I(P_2, \bar{\lambda}, t)} \gamma_{12}(t) \times s(r_1, \varphi_1, t) s(r_2, \varphi_2, t) & \end{aligned}$$

where A is the detector sensing area and $R(\bar{\lambda})$ is detector responsivity. Eq. (20) will be basis for obtaining expressions for optical tracker output signals. Let modified tracker be illustrated with Fig. 3, [3-7].

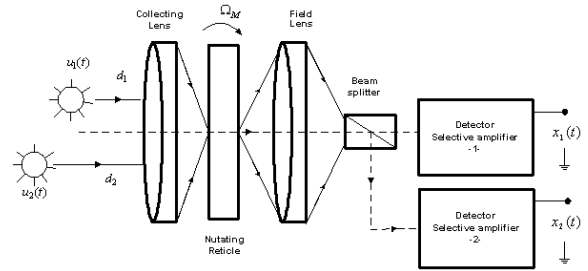


Figure 3. Modified optical tracker

The reason for using more detectors is inability of the existing tracker, Figure 1, to discriminate more optical sources, [3-13]. On the basis of Eq. (20) the photo-currents i_1 and i_2 can be obtained by simply inserting $\tau(\lambda)$ and $\rho(\lambda)$ in (20) where $\tau(\lambda)$ is the beam splitter transmission coefficient and $\rho(\lambda)$ is the beam splitter reflection coefficient. The optical tracker output signals x_1 and x_2 are obtained as:

$$x_j(t) = g_j(t) * i_j(t) \quad j \in \{1, 2\} \quad (21)$$

where g_1 and g_2 are impulse responses of the selective amplifiers and $*$ means temporal convolution. Based on Eq. (20) the following is obtained for quasi-monochromatic and monochromatic sources:

$$\begin{aligned} x_1(t) = g_{11}(t) * s(r_1, \varphi_1, t) + g_{12}(t) * s(r_2, \varphi_2, t) & \\ + g_{13}(t) * [\gamma_{12}(t) \times s(r_1, \varphi_1, t) s(r_2, \varphi_2, t)] & \\ x_2(t) = g_{21}(t) * s(r_1, \varphi_1, t) + g_{22}(t) * s(r_2, \varphi_2, t) & \\ + g_{23}(t) * [\gamma_{12}(t) \times s(r_1, \varphi_1, t) s(r_2, \varphi_2, t)] & \end{aligned} \quad (22)$$

where expressions for impulse responses are given in [4,5]. Due to the high level of non-stationarity we were not able to include in Eq. (22) the mutual degree of coherence $\gamma_{12}(t)$ as the part of the impulse responses.

The signal model Eq. (22) is reduced into linear one when optical sources are incoherent i.e. $\gamma_{12}(t) = 0$, [3-7].

Also, if $\gamma_{12}(t) = const.$ the signal model Eq.(22) is transformed into linear one by simple linear bandpass filtering, [4,5]. In the most general case when $\gamma_{12}(t)$ is some arbitrary function of time we can introduce additional source signal $s_3(t) = \text{Re}[\gamma_{12}(t)] \times s_1(t) s_2(t)$. We can either use ICA method developed for undercomplete representation, [26], in order to recover the three source signals from two measured signals or to introduce one additional beam splitter and one additional detector in order to recover the three unknown source signals on the basis of three measured signals. We can discard the source signal s_3 after recovery since we are not interested in it. Since the first two source signals are

sub-Gaussian signals the third source signal can be even Gaussian. The key point is that it must be statistically independent in relation to the first two source signals. Simulations show that second and fourth order cross-cumulants between s_1, s_2 and s_3 are more than 10 times smaller than related second and fourth order cumulants. One can also note that in the special case of quasimonochromatic radiation, Eq.(18), for $\bar{\lambda} = 1\mu\text{m}$ and relative velocity $v > 0.1 \text{ ms}^{-1}$ the numerical frequency $f = v/\bar{\lambda}$ is greater than 100 kHz in which case the nonlinear part in Eq.(22) is transformed on higher frequencies that are out of the pass-band region of the bandpass filters g_{13} and g_{23} so that signal model Eq.(22) could be again reduced on the linear one by simple linear bandpass filters.

4.0 Characterization of the Signal Model

We shall assume the linear form of the signal model (22) i.e. that optical sources are mutually incoherent with $\gamma_{12}(t) = 0$. There are two problems associated with the statistical inversion of the convolutive mixtures, Fig. 4 and Eq.(22): the whitening problems and problems with the non-minimum phase of the mixing system transfer function. The whitening problem can be solved by recurrent neural network architecture, Fig. 5, [14].

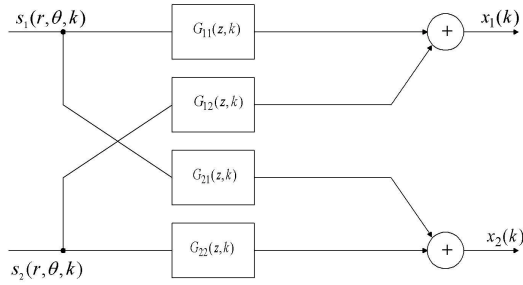


Figure 4. Convolutive signal model

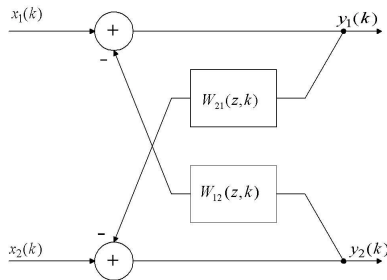


Figure 5. Recurrent separation network.

It is straightforward to derive relationships between the mixing filters and separation filters in Z domain:

$$\begin{aligned} W_{12}(z) &= -G_{12}(z)G_{11}(z)^{-1} \\ W_{21}(z) &= -G_{21}(z)G_{22}(z)^{-1} \end{aligned} \quad (23)$$

If mixing filters $G_{11}(z)$ and $G_{22}(z)$ have zeros outside the unit circle then non-causal realization of the separating filters $W_{12}(z)$ and $W_{21}(z)$ must be used in order to approximate unstable roots. Since any non-minimum phase system can be written as $G(z) = G_{\min}(z)G_{AP}(z)$, where $G_{\min}(z)$ is a minimum phase system and $G_{AP}(z)$ is an all-pass system, [24], the problem of inverting non-minimum phase system is to delay the inverting systems properly, [25]. For the recurrent

separation network such delay is obtained going to the frequency domain and performing signal separation on the block by block basis. Therefore, we have applied here an adaptive frequency domain algorithm, [23]. In order to identify the possible non-minimum phase problems we have applied the fourth-order cumulant based blind identification, [20], of the mixing filters $G_{11}(z)$ and $G_{22}(z)$ that were modeled as the FIR filters of the 14th order. Provided that input signals are non-Gaussian i.i.d. signals the coefficients of the FIR filter of the order L are obtained as [20]:

$$h(i) = \frac{C_4 y(L, 0, i)}{C_4 y(L, 0, 0)} \quad i=0, \dots, L \quad (25)$$

where $C_4 y(L, 0, i)$ are the fourth-order cumulants of the output signal y . Since in our case the input signal is FM signal, that belongs to the sub-Gaussian class of signals, the fourth-order cumulants exist. Figure 6. shows location of the zeros of such blindly identified FIR mixing filter. Obviously, there are zeros outside the unit circle. It should be noted that zero locations of the mixing filters $G_{11}(z)$ and $G_{22}(z)$ are influenced mainly by the character of the selective amplifiers impulse responses.

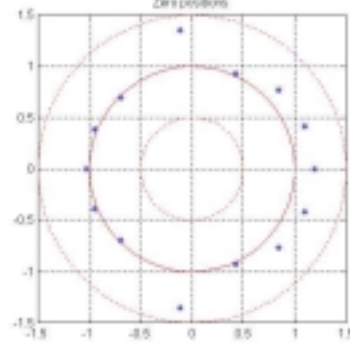


Figure 6. Zeros of the blindly identified FIR filter

The linear part of the convolutive signal model (22) can be transformed into frequency domain yielding:

$$\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \times \begin{bmatrix} S_1 \\ S_2 \end{bmatrix} \quad (26)$$

where all quantities in the Eq. (26) are Discrete Fourier Transforms (DFTs) of the related time domain quantities in the signal model (22). To recover the source signals we shall apply the slightly modified version of the adaptive frequency domain algorithm developed by Back and Tsoi, [22], that was itself the frequency domain extension of the time domain Herault-Jutten neural network [15,16]:

$$\begin{aligned} \Delta W_{ij} &= \Phi(\bar{y}_j, k) Y_i(k) \\ \Delta W_{ji} &= \Phi(y_i, k) Y_j^*(k) \end{aligned} \quad (27)$$

where $i=1, \dots, N-1$; $j=i+1, \dots, N$, N is the number of sources and:

$$\Phi(y_i) = STFT[\varphi(y_i)] \quad \Phi(\bar{y}_j) = STFT[\varphi(\bar{y}_j)]$$

$$y_i = ISTFT(Y_i) \quad \bar{y}_j = ISTFT(Y_j^*)$$

and nonlinearity $\varphi(\circ)$ is applied componentwise. The linear terms in Eq. (27) are due to the results developed by Amari, [21], for the instantaneous mixtures stating that learning rules will be super-efficient if only one odd nonlinear function is used. The closed form input-output relation of the recurrent neural network given with Fig. 5 is:

$$Y = (I + W)^{-1} X \quad (28)$$

where W is easily identified from Fig. 5. More details about this algorithm can be found in [23].

5.0 Experimental Results for the Incoherent (Heat) Sources

Measured signals x_1 and x_2 are obtained on the basis of the two frequency modulated (FM) source signals s_1 and s_2 by means of the optical tracking device, [3-7], whose schematic diagram is shown on the Figure 3. and photography of the working model is shown on the Figure 7. Deviation of the FM signal is proportional with the distance of the optical source from the optical axis of the optical tracker. Spectrogram of the measured signal x_1 is shown on Fig.8 and spectrogram of the measured signal x_2 looks similarly. It can be seen from the spectrogram that two signals, corresponding with associated optical sources, exist simultaneously in the measured signals x_1 and x_2 . When FM demodulator is applied on either signal x_1 or signal x_2 , only the IR optical source that was placed near the center of the filed of view (FOV) can be discriminated. If, however, the frequency domain based BSS algorithm, Eq. (27), is applied on the signals x_1 and x_2 the influence of the IR source placed near the center of the FOV can be eliminated and both IR sources can be discriminated. Here, we have used the filter length of 32 taps. In order to eliminate effects of the circular convolution 32 zeros are added to the signal vectors prior to doing FFT. So the overall FFT length was $L=64$. This FFT length introduces 1ms delay in the tracking loop making it suitable for real time tracking. We have done frequency domain implementation using overlap-save technique with overlap factor 0.5. The data were whitened before applying the BSS algorithm. Spectrograms of the output signals y_1 and y_2 , obtained according to Eq. (28) are shown on Fig. 9 and 10. It can be observed in signal y_1 that influence of the IR source placed near the center of the FOV is eliminated. Fig. 11 shows demodulated signals: the first one (with stable amplitude) obtained after demodulation of the original source signal and the second one obtained after demodulation of the recovered signal with $\varphi(y) = 2y + \text{sign}(y)y^2$.

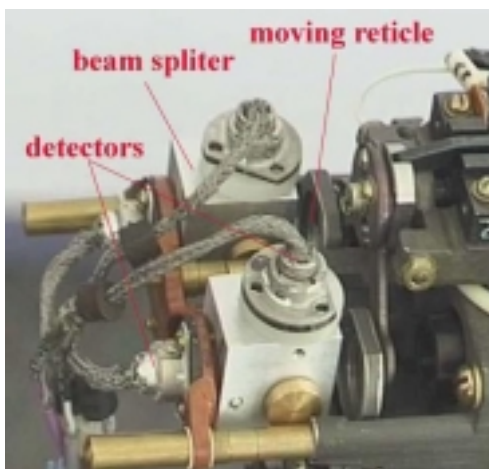


Figure 7. Functional model of the modified reticle tracker

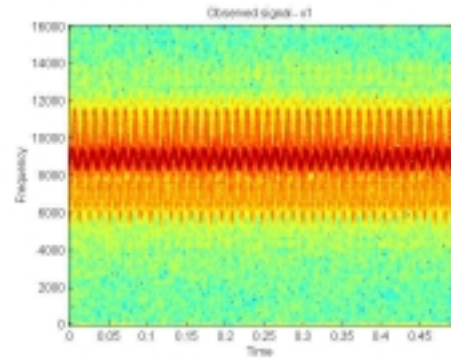


Fig.8 Spectrogram of the signal x_1

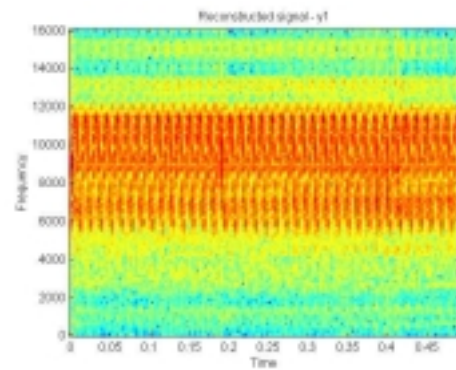


Fig.9 Spectrogram of the signal y_1

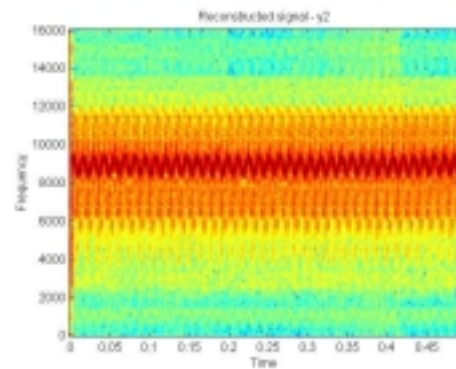


Fig.10 Spectrogram of the signal y_2

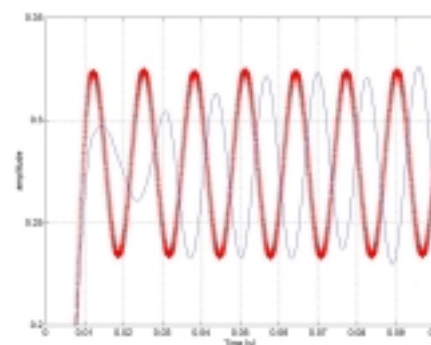


Figure 11. Demodulated signals: + original source; -- recovered source.

6.0 Conclusion

The beam splitter based modification of the reticle optical trackers is used for tracking and discrimination of the several optical sources. The mathematical framework called Independent Component Analysis (ICA) is used for that purpose. The theoretical basis of the problem is formulated by using statistical optics principles, partial coherence theory and Huygens-Fresnel principle. It has been shown analytically and verified experimentally that incoherent (heat) sources produce linear ICA model which enables the application of the linear ICA theory to recover the unknown reticle transmission functions (rtfs) that encode positions of the corresponding single optical sources. In a case of the partially coherent illumination by laser a nonlinear and highly nonstationary signal model is obtained. However, transformation into linear model is possible in special case when partially coherent optical sources are not in relative motion i.e. when the mutual degree of coherence is time invariant. If coherence factor is time dependent we can introduce additional source signal and apply linear ICA algorithms developed for undercomplete representation or additional sensor must be used in order to have the same number of sensors and sources.

7.0 References

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