UNIVERSAL FOURTH ORDER MUSIC METHOD : INCORPORATION OF ICA INTO MEG INVERSE SOLUTION

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ABSTRACT

In recent years, several inverse solutions of magnetoencephalography (MEG) have been proposed. Among them, the multiple signal classification (MUSIC) method utilizes spatio-temporal information obtained from magnetic fields. The conventional MUSIC method is, however, sensitive to Gaussian noise and a sufficiently large signal-to-noise ratio (SNR) is required to estimate the number of sources and to specify the precise locations of electrical neural activities. In this paper, a universal fourth order MUSIC (UFO-MUSIC) method, which is based on fourth order statistics, is proposed. This method is shown to be more robust against Gaussian noise than the conventional MUSIC method. It is an algebraic approach to independent component analysis (ICA). Although ICA and the analysis of the MEG inverse problem have been separately discussed, the proposed method incorporates ICA into the MEG inverse solution. The results of numerical simulations demonstrate the validity of the proposed method.

1. INTRODUCTION

Ionic currents associated with brain electrical activities generate weak magnetic fields around the head. These magnetic fields are called magnetoencephalograms (MEGs). Magnetoencephalography (MEG) has proven to be a useful noninvasive method for the localization of brain activities [1]. The magnetic fields generated from neural activities can be noninvasively measured by highly sensitive magnetic sensors called superconducting quantum interference devices (SQUIDs). The measured magnetic fields reflect the neural activity distribution and can be applied to obtain computer topographical mappings. In this magnetic inverse problem, magnetic sources are usually modeled by equivalent current dipoles. Although single dipole estimation is available for some clinical applications, multidipole estimation is commonly applied since it reveals more information about the complex neural activities.

Most multidipole estimation methods discuss how to optimize the dipole parameters so that the biomagnetic fields calculated from them sufficiently explain the measured data. Due to nonlinearity in the relationship, this parameter optimization is often difficult to perform. Moreover, the biomagnetic inverse problem is ill-posed.

Most of the inverse solutions are based on spatial information of magnetic fields that are measured at a particular time. Among them, several kinds of minimum norm estimations are well-known. There are also some solutions that utilize spatio-temporal information. The main assumption of the spatio-temporal model is that there are several dipolar sources that maintain their position and orientation, but vary only their magnitude as a function of time. Rather than fitting dipoles to measured magnetic fields from one instant in time, dipoles are fitted based on a certain time series.

The multiple signal classification (MUSIC) method [2] is one of the spatio-temporal MEG inverse solutions. It is based on the method originally developed in the field of sensor array processing [3]. The conventional MUSIC method uses only second order statistics and is based on principal component analysis (PCA). In this paper, we propose a new spatio-temporal MEG inverse solution, which takes advantage of fourth order statistics.

2. PROBLEM FORMULATION

The biomagnetic forward model, which describes the relationship between electrical sources in the brain and the measured magnetic fields above the head, can be formulated according to the *Biot-Sarvart law*. Assume that an array of n sensors receives q sources in the brain. Then, the forward model is written as

$$\boldsymbol{x}(t) = \boldsymbol{A}\boldsymbol{s}(t) + \boldsymbol{n}(t), \quad t = 1, \cdots, L,$$

where the n-dimensional vector $\boldsymbol{x}(t)$ describes the magnetic flux density at the measurement locations at time t, $\boldsymbol{A}$ is the $n \times q$ gain matrix, which describes the geometric relation between internal sources and external fields, the q-dimensional vector $\boldsymbol{s}(t)$ describes the internal current dipole moments, the n-dimensional vector $\boldsymbol{n}(t)$ describes sensor noise, and L is the number of time slices.

Due to the central limit theorem, the sensor noise is usually modeled as a Gaussian random process. It is often assumed to constitute a zero-mean, temporally and spatially white Gaussian random process. Moreover, we assume that the sources and noise are stationary and statistically independent; the time series for different source components are asynchronous or linearly independent.

3. MUSIC METHOD

The MUSIC method provides a solution for the above equation; it specifies the number of sources and parameters which describe the locations of each source in the brain. In this method, the electrical source distribution in the brain is assumed to be locally modeled as a current dipole. The MUSIC method is largely dependent on second order statistics of the spatio-temporally measured magnetic fields. The covariance matrix of sensor outputs is given by

$$\boldsymbol{R} = E[\boldsymbol{x}\boldsymbol{x}^T] = \boldsymbol{A}\boldsymbol{S}\boldsymbol{A}^T + \sigma^2\boldsymbol{I},$$

where $\boldsymbol{S} = E[\boldsymbol{s}\boldsymbol{s}^T]$ is the $q \times q$ covariance matrix of sources, σ^2 is the noise covariance, and $E[\cdot]$, $\boldsymbol{B}^T$ and $\boldsymbol{I}$ denote the expectation operator, transpose of a matrix $\boldsymbol{B}$, and the identity matrix, respectively. It can be eigendecomposed as

$$\boldsymbol{R} = \boldsymbol{U}_S\boldsymbol{\Lambda}_S\boldsymbol{U}_S{}^T + \sigma^2\boldsymbol{U}_N\boldsymbol{U}_N{}^T,$$

where the $q \times q$ diagonal matrix $\boldsymbol{\Lambda}_S$ contains the q largest eigenvalues in descending order, and each column vector of the $n \times q$ matrix $\boldsymbol{U}_S$ is the corresponding signal eigenvector. Similarly, the $n \times (n-q)$ matrix $\boldsymbol{U}_N$ contains the $n-q$ noise eigenvectors that correspond to the noise eigenvalue σ^2. The spaces spanned by $\boldsymbol{U}_S$ and $\boldsymbol{U}_N$ are referred to as the *signal subspace* and the *noise subspace*, respectively. Consistent estimates of eigenvalues and eigenvectors can be found using the eigendecomposition of the sample covariance matrix

$$\hat{\boldsymbol{R}} = \frac{1}{L}\sum_{t=1}^{L}\boldsymbol{x}(t)\boldsymbol{x}^T(t) = \hat{\boldsymbol{U}}_S\hat{\boldsymbol{\Lambda}}_S\hat{\boldsymbol{U}}_S^T + \hat{\boldsymbol{U}}_N\hat{\boldsymbol{\Lambda}}_N\hat{\boldsymbol{U}}_N^T,$$

where the $\hat{q} \times \hat{q}$ and $(n-\hat{q}) \times (n-\hat{q})$ diagonal matrices $\hat{\boldsymbol{\Lambda}}_S$ and $\hat{\boldsymbol{\Lambda}}_N$ contain the $\hat{q}$ and $n-\hat{q}$ signal and noise eigenvalues, respectively, the column vectors of the $n \times \hat{q}$ and $n \times (n-\hat{q})$ matrices $\hat{\boldsymbol{U}}_S$ and $\hat{\boldsymbol{U}}_N$ contain the corresponding eigenvectors, and $\hat{q}$ denotes any consistent estimate of the number of sources.

Based on these observations, the cost function

$$f_{MUSIC}(i) \quad = \quad \lambda_{min}\{G_i{}^T\hat{\boldsymbol{U}}_N\hat{\boldsymbol{U}}_N^T G_i\} \qquad (1)$$

becomes zero for any gain matrix $\boldsymbol{A}$ corresponding to a true dipole location, where i denotes a grid point, G_i describes the principal left eigenvectors of the matrix $\boldsymbol{A}$, and $\lambda_{min}\{\boldsymbol{B}\}$ is a minimum eigenvalue of a matrix $\boldsymbol{B}$. Thus, the cost function (1) can be used as the criterion for source existence. In the MUSIC algorithm, we calculate (1) for each grid, which is set over two-dimensional slices through the three-dimensional space, e.g., xy-planes for constant z, and plot the function $1/f_{MUSIC}(i)$.

After placing $\hat{q}$ dipoles at each peak, we can easily determine the direction of each dipole and its magnitude by solving linear optimization problems.

4. UFO-MUSIC METHOD

Although the expansion of the MUSIC method, e.g., the recursively applied and projected MUSIC (RAP-MUSIC) method [4], represents significant advances in source localization, it still fails to recognize the unreliability of multiple dipole source solutions as compared to single dipole source solutions. These solutions are very sensitive to noise, which limits the usefulness of these solutions from a clinical point of view.

Since the measured magnetic fields are generated by electrical sources that generally conform to non-Gaussian random processes, higher order methods are usually preferred to conventional second order algorithms such as the MUSIC algorithm. In contrast to the conventional MUSIC method, the universal fourth order MUSIC (UFO-MUSIC) method utilizes higher order statistics, i.e., fourth order statistics. The main advantage of this method is its improved robustness against arbitrary Gaussian noise. Also, from a computational point of view, the MUSIC-like estimators based on *contracted quadricovariance* provide high quality performance at moderate computational costs [5].

We assumed that the sources were non-Gaussian circular processes and the noise was a colored Gaussian process with an unknown covariance matrix. The quadricovariance is defined as the set of fourth order cumulants that can be written under the circularity assumption as

$$\begin{aligned} cum(x_i, x_j, x_k, x_l) \quad = \quad & \mu_4(x_i, x_j, x_k, x_l) \\ & - \mu_2(x_i, x_j)\mu_2(x_k, x_l) \\ & - \mu_2(x_i, x_l)\mu_2(x_k, x_j), \\ & 1 \le i, j, k, l \le n, \quad (2) \end{aligned}$$

where second and fourth order moments are defined respectively as

$$\mu_2(x_i, x_j) = E[x_i x_j], \quad 1 \le i, j \le n,$$
$$\mu_4(x_i, x_j, x_k, x_l) = E[x_i x_j x_k x_l], \quad 1 \le i, j, k, l \le n.$$

The contracted quadricovariance is defined as

$$[\boldsymbol{Q}(\boldsymbol{M})]_{ij} = \sum_{k,l=1}^{n} cum(x_i, x_j, x_k, x_l)\ m_{kl}, \quad 1 \le i, j \le n, \quad (3)$$

where each m_{kl} is the entry of a free $n \times n$ matrix $\boldsymbol{M}$. Using Eq. (2), we rewrite it in a matrix form as

$$\boldsymbol{Q}(\boldsymbol{M}) = E[\boldsymbol{x}^T \boldsymbol{M} \boldsymbol{x} \boldsymbol{x} \boldsymbol{x}^T] - \boldsymbol{R} \boldsymbol{M} \boldsymbol{R} - Tr(\boldsymbol{M} \boldsymbol{R}) \boldsymbol{R}, (4)$$

where $Tr(\boldsymbol{B})$ denotes the trace of a matrix $\boldsymbol{B}$. We call $\boldsymbol{Q}(\boldsymbol{M})$ cumulant matrix of any $n \times n$ matrix $\boldsymbol{M}$. The sample estimate of this matrix can be expressed as

$$\hat{\boldsymbol{Q}}(\boldsymbol{M}) = \frac{1}{L} \sum_{t=1}^{L} \boldsymbol{x}^T(t) \boldsymbol{M} \boldsymbol{x}(t) \boldsymbol{x}(t) \boldsymbol{x}^T(t) - \hat{\boldsymbol{R}} \boldsymbol{M} \hat{\boldsymbol{R}} - Tr(\boldsymbol{M} \hat{\boldsymbol{R}}) \hat{\boldsymbol{R}}.$$

The eigendecomposition of (4) theoretically yields

$$\boldsymbol{Q}(\boldsymbol{M}) = \boldsymbol{E}_S \boldsymbol{\Gamma}_S \boldsymbol{E}_S^T,$$

where the diagonal matrix $\boldsymbol{\Gamma}_S$ contains the q nonzero eigenvalues of the highest magnitude in descending order, and each column vector of the $n \times q$ matrix $\boldsymbol{E}_S$ is the corresponding signal eigenvector. Under the assumption that the sensor noise conforms to Gaussian distribution and that the sources and noise are statistically independent, the noise eigenvalues of $\boldsymbol{\Gamma}_N$ are zero; the term $\boldsymbol{\Gamma}_N$ disappears.

Consistent estimates of eigenvalues and eigenvectors can be found using the eigendecomposition of the sample contracted quadricovariance matrix

$$\hat{\boldsymbol{Q}}(\boldsymbol{M}) = \hat{\boldsymbol{E}}_S \hat{\boldsymbol{\Gamma}}_S \hat{\boldsymbol{E}}_S^T + \hat{\boldsymbol{E}}_N \hat{\boldsymbol{\Gamma}}_N \hat{\boldsymbol{E}}_N^T.$$

Similar to (1), the contracted quadricovariance-based MUSIC-like estimator is given by

$$f_{UFO-MUSIC}(i) = \lambda_{min}\{G_i{}^T \hat{\boldsymbol{E}}_N \hat{\boldsymbol{E}}_N^T G_i\}, \quad (5)$$

where $\hat{\boldsymbol{E}}_N$ is the $n \times (n - \hat{q})$ matrix of the noise subspace eigenvectors associated with $n - \hat{q}$ eigenvalues of the smallest magnitude.

The simplest way to select a cumulant matrix $\boldsymbol{Q}(\boldsymbol{M})$ is to assume $\boldsymbol{M} = \boldsymbol{I}$. In this case, (4) is simplified as

$$\hat{\boldsymbol{Q}}(\boldsymbol{I}) = \frac{1}{L} \sum_{t=1}^{L} \boldsymbol{x}^T(t) \boldsymbol{x}(t) \boldsymbol{x}(t) \boldsymbol{x}^T(t) - \hat{\boldsymbol{R}}^2 - Tr(\hat{\boldsymbol{R}}) \hat{\boldsymbol{R}}.$$

Interestingly, the eigenvalues of this matrix correspond to *kurtosis* of each signal. This simple choice, however, results in the loss of information contained in the other cumulants; the above case uses only a fraction of the fourth order information. In Eq. (3), m_{kl} represents the weight of $cum(x_i, x_j, x_k, x_l)$. Therefore, m_{kl} determines the weight of the information that each $cum(x_i, x_j, x_k, x_l)$ contains. In order to exploit as much information from the cumulants as possible, several matrices $\boldsymbol{M}$ should be chosen. Also it is desirable to estimate more plausible signal subspace and noise subspace by diagonalizing several matrices $\boldsymbol{Q}(\boldsymbol{M})$ simultaneously. This can be performed by applying the joint diagonalization method.

5. JOINT DIAGONALIZATION AND SELECTION OF CUMULANT MATRICES

In this section, we introduce an important criterion based on the cumulant to effectively separate the space into the signal subspace and the noise subspace [6]-[8].

5.1. Optimization of cumulant criterion

The (k, l)-th cumulant slice is defined as the matrix whose (i, j)-th entry is $cum(i, j, k, l)$. It is equal to $\boldsymbol{Q}(\boldsymbol{M})$ by assuming that $\boldsymbol{M} = \boldsymbol{b}_k \boldsymbol{b}_l^T$, where $\boldsymbol{b}_k$ denotes the n-dimensional vector with 1 in k-th position and 0 elsewhere. Note that a cumulant matrix $\boldsymbol{Q}(\boldsymbol{M})$ may represent a linear combination of parallel cumulant slices with the entries of $\boldsymbol{M}$ as coefficients. Here, we define the parallel set $\mathcal{N}^p$ as the set of all the parallel slices

$$\mathcal{N}^p = \{\boldsymbol{Q}(\boldsymbol{b}_k \boldsymbol{b}_l^T) | 1 \le k, l \le n\}.$$

Using the cumulant properties, it is straightforwardly established that

$$\boldsymbol{Q}(\boldsymbol{M}) = \boldsymbol{U} \boldsymbol{\Lambda}_M \boldsymbol{U}^T, \quad (6)$$
$$\boldsymbol{\Lambda}_M = diag(\lambda_1 \boldsymbol{u}_1^T \boldsymbol{M} \boldsymbol{u}_1, \ldots, \lambda_n \boldsymbol{u}_n^T \boldsymbol{M} \boldsymbol{u}_n),$$

where λ_i denotes $cum(s_i, s_i, s_i, s_i)$ which means the cumulant of source $\boldsymbol{s}$, and $\boldsymbol{u}_i$ is the i-th column vector of $\boldsymbol{U}$. Thus, matrix $\boldsymbol{U}$ diagonalizes $\boldsymbol{Q}(\boldsymbol{M})$ and the column vectors of $\boldsymbol{U}$ can be identified to the eigenvectors of $\boldsymbol{Q}(\boldsymbol{M})$ for any matrix $\boldsymbol{M}$.

Let $\boldsymbol{V}$ denote a $n \times n$ unitary matrix and further define $\boldsymbol{e}(t)$ as

$$\boldsymbol{e}(t) = \boldsymbol{V}^T \boldsymbol{x}(t).$$

If $\boldsymbol{V} = \boldsymbol{U}$ then $\boldsymbol{V}^T\boldsymbol{U} = \boldsymbol{I}$ and the coordinates of $\boldsymbol{e}(t)$ are the noise corrupted sources.

It is desirable to determine $\boldsymbol{U}$ as the unitary minimizer of the sum of all the squared cross-cumulants. Since the sum of the squared cross-cumulants plus the sum of the squared auto-cumulants does not depend on $\boldsymbol{V}$ as long as $\boldsymbol{V}$ remains unitary, this is equivalent to maximizing the criterion

$$c'(\boldsymbol{V}) = \sum_{i=1}^{n} |cum(e_i, e_i, e_i, e_i)|^2.$$

We attempt to determine $\boldsymbol{U}$ as the symmetry maximizer of the criterion

$$c(\boldsymbol{V}) \quad = \quad \sum_{i,j,k=1}^{n} |cum(e_i, e_i, e_j, e_k)|^2, \tag{7}$$

which is equivalent to minimizing the sum of squared cross-cumulants with distinct first and second indices. The main reason for considering this criterion is that it is related to an underlying eigenstructure, which allows efficient optimization by *joint diagonalization.*

5.2. Joint diagonalization

Let $\mathcal{N} = \{\boldsymbol{N}_r | 1 \le r \le s\}$ represent a set of s matrices with a common size $n \times n$. A joint diagonalizer of the set $\mathcal{N}$ is defined as a unitary maximizer of the criterion

$$C(\boldsymbol{V}, \mathcal{N}) = \sum_{r=1}^{s} |diag(\boldsymbol{V}^T \boldsymbol{N}_r \boldsymbol{V})|^2, \tag{8}$$

where $|diag(\boldsymbol{B})|^2$ is the squared sum of diagonal elements of a matrix $\boldsymbol{B}$. If the set $\mathcal{N}$ cannot be exactly jointly diagonalized, the unitary maximization of (8) defines somewhat arbitrary but quite natural joint approximate diagonalization. According to [6], we have

$$c(\boldsymbol{V}) = C(\boldsymbol{V}, \mathcal{N}^p)$$

for any unitary matrix $\boldsymbol{V}$. Thus, the unitary maximization of $c(\boldsymbol{V})$ is equivalent to the joint diagonalization of the parallel set.

5.3. A new method to select cumulant matrices

Although the construction of $\boldsymbol{Q}(\boldsymbol{M})$ by assuming $\boldsymbol{M} = \boldsymbol{b}_k \boldsymbol{b}_l^T$ for all $1 \le k, l \le n$ is ideal, joint diagonalization of n^2 of $\boldsymbol{Q}(\boldsymbol{M})$ is practically impossible. Here, we attempt to increase the computational efficiency of joint diagonalization by selecting p appropriate cumulant matrices.

As an orthonormal basis which spans the n^2-dimensional space, we consider

$$\boldsymbol{M} = \begin{cases} \boldsymbol{b}_k \boldsymbol{b}_l^T, & k = l, \\ \dfrac{\boldsymbol{b}_k \boldsymbol{b}_l^T + \boldsymbol{b}_l \boldsymbol{b}_k^T}{\sqrt{2}}, & k < l, \\ \dfrac{\boldsymbol{b}_k \boldsymbol{b}_l^T - \boldsymbol{b}_l \boldsymbol{b}_k^T}{\sqrt{2}}, & k > l. \end{cases}$$

Hence,

$$\boldsymbol{Q}(\boldsymbol{M}) = \begin{cases} \boldsymbol{Q}(\boldsymbol{b}_k \boldsymbol{b}_l^T), & k = l, \\ \sqrt{2}\, \boldsymbol{Q}(\boldsymbol{b}_k \boldsymbol{b}_l^T), & k < l, \\ 0, & k > l. \end{cases}$$

It is not necessary to compute the cumulant matrices $\boldsymbol{Q}(\boldsymbol{M})$ for $k > l$. It is sufficient to estimate and to diagonalize $n(n+1)/2$ cumulant matrices.

However, it is still difficult to jointly diagonalize $n(n+1)/2$ cumulant matrices. Since $[\boldsymbol{Q}(\boldsymbol{b}_k \boldsymbol{b}_l^T)]_{ij} = cum(x_i, x_j, x_k, x_l)$ holds, we select P cumulant matrices $\boldsymbol{Q}(\boldsymbol{M}_p), 1 \le p \le P$ such that the squared sum of its diagonal elements has the largest values. This method facilitates joint diagonalization of as many cumulant matrices as possible. In particular, when measured signals are nearly independent, jointly diagonalizing cumulant matrices $\boldsymbol{Q}(\boldsymbol{M}_p)$ for each p is equivalent to maximizing the criterion (7); the parameter P determines the extent of maximization.

5.4. Summary of UFO-MUSIC algorithm

We attempt to separate the space into the signal subspace and the noise subspace utilizing joint diagonalization. The separation process is summarized as follows:

1. Calculate $n(n+1)/2$ matrices $\hat{\boldsymbol{Q}}(\boldsymbol{M})$ such that

$$\boldsymbol{M} = \begin{cases} \boldsymbol{b}_k \boldsymbol{b}_l^T, & k = l, \\ \dfrac{\boldsymbol{b}_k \boldsymbol{b}_l^T + \boldsymbol{b}_l \boldsymbol{b}_k^T}{\sqrt{2}}, & k < l. \end{cases}$$

2. Select P cumulant matrices $\hat{Q}(\boldsymbol{M}_p)$, $1 \le p \le P$ such that the squared sum of its diagonal elements has the largest values.

3. Jointly diagonalize $\hat{Q}(\boldsymbol{M}_p)$ for each $1 \le p \le P$ and find the matrix $\boldsymbol{V}$.

Furthermore, to separate the matrix $\boldsymbol{V}$ into the signal eigenvectors and the noise eigenvectors, we determine averaged eigenvalues

$$\xi_i = \frac{1}{n}\sum_{p=1}^{n} \lambda_{pi}, \quad 1 \le i \le n,$$

where λ_{pi} denotes the eigenvalues of $\hat{\boldsymbol{Q}}(\boldsymbol{M}_p)$.

4. Array the eigenvalues ξ_i in descending order and select corresponding $n - \hat{q}$ noise eigenvectors to $n - \hat{q}$ eigenvalues of the smallest magnitude.

5. Form $\hat{\boldsymbol{E}}_N$ and calculate (5) at each grid.

6. SIMULATION RESULTS AND DISCUSSION

Volume currents were explicitly not accounted for in the conductivity geometry. This assumption is valid, for instance, in the case of a conducting half-space, where magnetic conductivity is a function of the z coordinate only. Our choice of conductivity distribution is for convenience only. Point magnetometers, measuring the z-component of the magnetic field, were assumed in the simulations.

Once the arrangement of the sensors were chosen, the signal in each magnetometer was calculated from test current dipoles. Normally distributed random numbers, representing a specified level of white noise, were added to each signal.

In the set of simulations, 36 point magnetometers were placed in a planar square lattice to cover an area of 80×80 mm^2 in the $z = 30$ mm plane. Thus, the distance between neighboring channels was 16 mm. The source current was assumed to be confined to a square of 200×200 mm^2 in the xy-plane, 30 mm from the magnetometer plane.

In the first simulation, the test source was a single current dipole that was directed in parallel to the y-axis, and whose magnitude changes conformed to the fifth order Daubechies' wavelet function. It was placed in the center of the xy-plane.

In the second simulation, the test sources were two current dipoles that were also directed in parallel to the y-axis. The magnitude changes of the two dipoles conformed to the fifth and fourth order Daubechies' wavelet functions. The two dipoles were placed symmetrically on the xy-plane.

We adopted wavelet functions as time-varying functions of the source's magnitude since such kinds of functions aptly describe the time variation of action potentials in the brain.

Figures 1 and 2 show the measured magnetic fields by all magnetometers in the first and second simulation, respectively. We analysed these data using the conventional MUSIC method and the proposed UFO-MUSIC method. The number of scanning grids of both the MUSIC and UFO-MUSIC methods was 121. All the grids were placed in a planar square lattice to cover an area of 200×200 mm^2 in the xy-plane. Thus, the distance between neighboring grids was 20 mm. Moreover, the threshold of the eigenvalue,

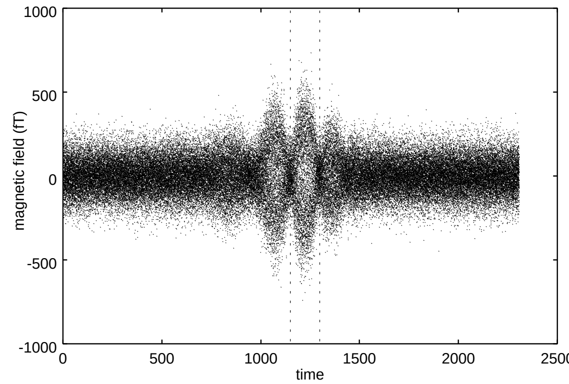

Fig. 1. Magnetic fields measured by all magnetometers in the first simulation. SNR = -3.34 (dB). The sampled interval is represented in the area between the dotted lines.

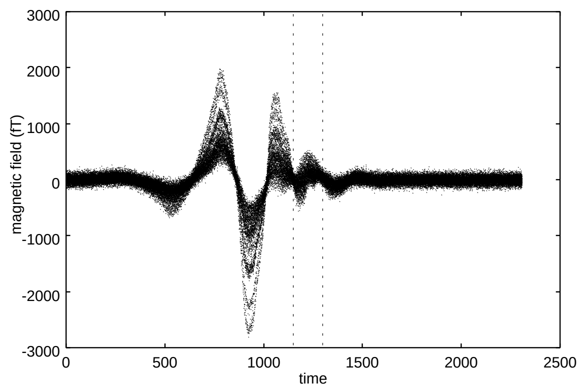

Fig. 2. Magnetic fields measured by all magnetometers in the second simulation. SNR = 4.63 (dB). The sampled interval is represented in the area between the dotted lines.

which separates the signal eigenvalues and noise eigenvalues, was set at 1/100 of the largest eigenvalue in both methods.

The results of the first and second simulation are shown in Figs. 3 and 4, respectively. The estimated number of sources increased as the signal-to-noise ratio (SNR) decreased in both methods, but the UFO-MUSIC method estimated the number more precisely than the MUSIC method, even in the case of low SNR. The locations of sources were correctly specified in both methods when the number of sources was precisely estimated.

The simulation results reveal that the UFO-MUSIC method is more robust against Gaussian noise than the conventional MUSIC method.

In this paper, we assumed that the sources were statistically independent; the time series for different source components were asynchronous or linearly independent. In

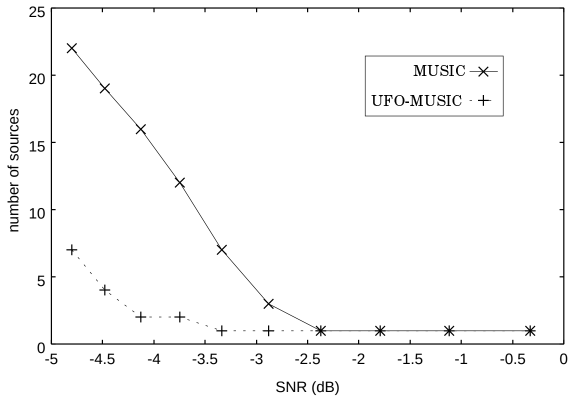

Fig. 3. Estimated number of sources in the first simulation

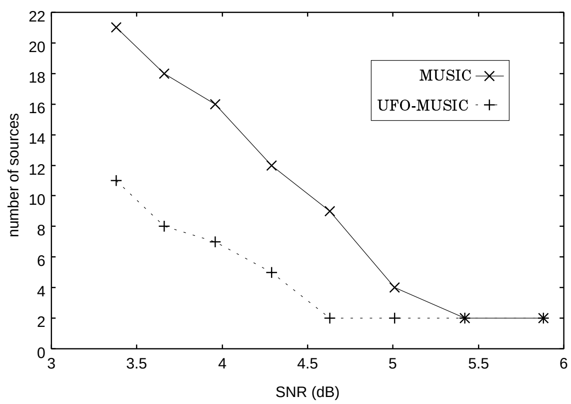

Fig. 4. Estimated number of sources in the second simulation

such a case, the signal subspace and the noise subspace can be separated by joint diagonalization. The UFO-MUSIC method is an algebraic approach to a conventional preprocessor of MEG data analysis, independent component analysis (ICA) [7], [8]. Although ICA and the analysis of the MEG inverse problem have been separately discussed [9], [10] , the proposed method incorporates ICA into the MEG inverse solution.

However, when sources are correlated or synchronous, the proposed UFO-MUSIC may have to be expanded to a RAP-MUSIC-like method [4]. Further studies will clarify how to approach such a case.

7. ACKNOWLEDGEMENT

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