BLIND SOURCE SEPARATION WITH PURE DELAY MIXTURES

Arie Yeredor

Department of Electrical Engineering - Systems
Tel-Aviv University, Tel-Aviv, 69978, ISRAEL
e-mail: `arie@eng.tau.ac.il`

ABSTRACT

We address the problem of blind separation of mixtures consisting of pure unknown delays in addition to scalar mixing coefficients. Such a mixture is a hybrid situation resembling both static and convolutive mixtures, but essentially different from both: On one hand, static-mixture approaches cannot be readily applied in this context; On the other hand, conventional convolutive-mixture approaches are not only over - parameterized for this problem, but are also incapable of accommodating fractional delays. We propose a second-order statistics based algorithm, which uses a specially parameterized approximate joint diagonalization of spectral matrices to estimate the mixing coefficients as well as the delays. The joint diagonalization algorithm is an extension of the "AC-DC" algorithm, previously proposed in the context of static mixtures. We provide analytic expressions for all minimization steps for the two sensors / two sources case, and demonstrate the performance using simulations results.

1. INTRODUCTION

Traditionally, blind source separation has been addressed in two distinct contexts: static mixtures and convolutive mixtures. In the static mixture context (e.g., [1] and references therein) it is assumed that a constant memoryless mixing matrix relates the source signals to the observed signals. In the convolutive (or dynamic) mixture context (e.g., [2, 3, 4]) it is assumed that each of the observed signals consists of differently filtered combinations of the source signals. The different mixture filters are usually assumed to be Finite Impulse Response (FIR), but are otherwise unconstrained. Naturally, a static mixture is a special case of the more general convolutive mixture, when the FIR length is one.

In this paper we address an interim situation, where the mixing is constrained to consist of pure delays in addition to static mixture coefficients. The delays are assumed to have occurred prior to the sampling process, and are therefore not necessarily an integer multiple of the sampling period. The (pre-sampled, continuous-time) source signals are assumed to be band-limited, so that the sampling is at least at the Nyquist rate (i.e., the sampling rate is at least twice the maximal frequency of the signals' spectra). Although such delays can be represented as convolutive mixtures in discrete-time, such a representation requires an infinite number of coefficients, and cannot consist of causal FIR models. Thus, this framework cannot be regarded as a special case of the (FIR) convolutive-mixture framework. The only parameters to be estimated in addition to the static mixture coefficients are the delays, rather than multiple FIR coefficients.

As customary in the "blind" framework, no prior knowledge is assumed on the mixing. As for the sources, we shall only assume that they are statistically independent wide-sense stationary (WSS) random signals with unknown spectra, properly band-limited as discussed above. In fact, since our estimation procedure will be based on second-order statistics only, it would suffice to assume that the sources are uncorrelated, which is a weaker assumption than statistical independence.

The occurrence of pure delays mixtures in practice is likely in situations of sensor arrays positioned in non-reverberant environments, such as in open-space acoustics situation. Nevertheless, the problem of estimating and separating pure-delays mixtures has seen little treatment in the literature so far. In [5] some preliminary analytic solutions are proposed, based on second and/or fourth order spectra; However, the model addressed does not allow for unknown mixing coefficients in addition to the delays (although it is indicated that one of the solutions can be adapted to accommodate ones). In [6] the case of fewer sensors than sources is addressed, but again there's no provision for unknown mixing coefficients. In [7] the delays are assumed to be integer multiples of the sampling period.

In this paper we address the following L sources -

L sensors model (to be later reduced to $L = 2$):

$$x_p(t) = \sum_{q=1}^{L} a_{pq} s_n(t - \tau_{pq}) \quad p = 1, 2, \ldots L \qquad (1)$$

where a_{pq} are the mixing coefficients and τ_{pq} are the delays from source q to sensor p. To mitigate the ambiguity associated with the sources' undetermined time-origin, we use as a "working assumption" zero delays from each source to the "respective" sensor, i.e., $\tau_{pp} = 0$ for $p = 1, 2, \ldots L$. Additional ambiguities are associated with the possible commutation of scales between the sources and the mixing coefficients. These can be resolved, e.g., by assuming that all sources have unit power, but such an assumption is immaterial to the algorithm derived in here.

The available data are samples of the continuous-time observations,

$$x_p[n] = x_p(nT) \quad n = 1, 2, \ldots N \qquad (2)$$

where T is the sampling period (we shall use parentheses / brackets to enclose continuous / discrete indices, respectively). As mentioned above, it is assumed that all the source signals (and hence the observed signals) are WSS with unknown spectra, bandlimited at (angular) frequency π/T. For simplicity, we also assume that the sources have zero mean. From the observed samples it is desired to estimate the mixing coefficients and delays, and to recover the (sampled, possibly scaled and delayed version of) the source signals.

The paper is organized as follows: In the next section we present the estimation problem as a specially parameterized joint diagonalization problem in the frequency domain; In section 3 we propose an iterative algorithm for the joint diagonalization, based on an extension of the "AC-DC" algorithm [8]; In section 4 we address the reconstruction of source signals; In section 5 we present some simulations results, with concluding remarks in section 6.

2. FORMULATION AS A JOINT DIAGONALIZATION PROBLEM

Since we assumed that the source signals are zero-mean WSS, the received signals' correlation functions are given (using (1) and the sources' statistical independence) by

$$\begin{aligned} R^x_{mn}(\tau) &= E[x_m(t+\tau)x_n(t)] \\ &= \sum_{p=1}^{L}\sum_{q=1}^{L} a_{mp}a_{nq} E[s_p(t-\tau_{mp}+\tau)s_q(t-\tau_{nq})] \\ &= \sum_{q=1}^{L} a_{mq}a_{nq} R^s_q(\tau+\tau_{nq}-\tau_{mq}) \quad 1 \le m, n \le L \end{aligned} \qquad (3)$$

where $R^x_{mn}(\tau)$ denotes the correlation between the m-th and the n-th received signals, and $R^s_q(\tau)$ denotes the autocorrelation of the q-th source signal.

Fourier-transforming (3), we obtain the relations between cross spectra:

$$S^x_{mn}(\omega) = \sum_{q=1}^{L} a_{mq}a_{nq} S^s_q(\omega) e^{-j\omega(\tau_{mq}-\tau_{nq})} \qquad 1 \le m, n \le L \quad (4)$$

where $S^x_{mn}(\omega)$ is the cross-spectrum between the m-th and n-th received signal and $S^s_q(\omega)$ is the q-th source's (unknown) spectrum. Eq. (4) can also be expressed in matrix-form as

$$\boldsymbol{S}_x(\omega) = \boldsymbol{B}(\omega)\boldsymbol{S}_s(\omega)\boldsymbol{B}^H(\omega) \qquad (5)$$

where $\boldsymbol{S}_x(\omega)$ is an $L \times L$ matrix consisting of $S^x_{mn}(\omega)$ as the m, n-th element, $\boldsymbol{S}_s(\omega)$ is an $L \times L$ diagonal matrix consisting of $S^s_q(\omega)$ as its q, q-th elements, and $\boldsymbol{B}(\omega)$ is the $L \times L$ matrix given by

$$\boldsymbol{B}(\omega) = \boldsymbol{A} \odot \boldsymbol{D}(\omega) \qquad (6)$$

where $\odot$ denotes Hadamard's (element-wise) product, $\boldsymbol{A}$ is the constant matrix of mixing coefficients, whose m, n-th element is a_{mn}, and $\boldsymbol{D}(\omega)$ contains the delays, such that its m, n-th element is given by

$$D_{mn} = e^{-j\omega\tau_{mn}} \quad 1 \le m, n \le L. \qquad (7)$$

If the cross-spectral matrices $\boldsymbol{S}_x(\omega)$ were known, then by using (5) at several frequencies $\omega_0, \omega_1, \ldots \omega_K$, a system of nonlinear equations could be formed, warranting an exact solution for the unknown mixing parameters and sources spectra (under some regularity conditions). In practice, however, these matrices are unknown, but can be estimated from the available data.

A possible strategy for non-parametric estimates of these matrices would be to use the Discrete-Time Fourier Transform (DTFT) of a truncated series of unbiased cross-correlations estimates (Blackman-Tuckey's method, e.g., [9]). Specifically, to estimate the m, n-th element of $\boldsymbol{S}_x(\omega)$, the following can be used:

$$\hat{S}^x_{mn}(\omega) = \sum_{l=-M}^{M} \hat{R}_{mn}[l] e^{-j\omega l} \qquad (8)$$

where

$$\hat{R}_{mn}[l] = \frac{1}{N-|l|} \sum_{p=1}^{N-|l|} x_m[p+l]x_n[p] \quad -M \le l \le M. \qquad (9)$$

and M is the truncation-window length. If M is larger than the sum of the longest correlation length (among all source signals) and the maximal delay, then these are unbiased estimates of the desired (cross-) spectra.

Note that in the transition from continuous time to discrete time, the frequency axis is rescaled to the range $-\pi : \pi$, resulting in some constant (and irrelevant) scaling of the estimated spectra. However, if, as assumed earlier, the sampling rate is higher than the Nyquist rate, then there's no loss of information, and the only distortion is in the scaling. Consequently, the estimated delays will later have to be translated from sample units to time units via multiplication by T.

When estimated values, rather than true values, of $\boldsymbol{S}_x(\omega)$ are used, the equations (5) usually can no longer be satisfied simultaneously at all frequencies. Nevertheless, once $\boldsymbol{S}_x(\omega)$ is estimated at several frequencies $\omega_0, \omega_1, \ldots \omega_K$, an estimate of the unknown parameters of interest can be obtained by resorting to approximate joint diagonalization (see e.g. [8, 4]), seeking to minimize the following least-squares (LS) criterion:

$$\min_{\boldsymbol{A},\boldsymbol{T},\boldsymbol{\Gamma}} C_{LS} \stackrel{\triangle}{=} \sum_{k=0}^{K} ||\boldsymbol{S}_x(\omega_k) - \boldsymbol{B}(\omega_k)\boldsymbol{S}_s(\omega_k)\boldsymbol{B}^H(\omega_k)||_F^2 \tag{10}$$

where $\boldsymbol{T}$ is an $L \times L$ matrix containing the delay parameters τ_{mn}, $\boldsymbol{\Gamma}$ is an $L \times (K+1)$ matrix containing the sources' spectra,

$$\gamma_{mk} = S_m^s(\omega_k) \quad 1 \leq m \leq L \quad 0 \leq k \leq K \tag{11}$$

and $|| \cdot ||_F^2$ denotes the squared Frobenius norm. Note that it is also possible to use a weighted LS criterion by introducing some positive weights w_k into the sum; however, to simplify the exposition, we shall not pursue this possibility in here.

Several algorithms exist for joint diagonalization of sets of matrices. However, these algorithms assume a fixed diagonalizing matrix $\boldsymbol{B}$, rather than $\boldsymbol{B}(\omega_k)$ which depends on the index k. In the next section we propose a modification (actually an extension) of an existing joint diagonalization algorithm, namely the AC-DC algorithm [8], adapted to this minimization problem.

3. JOINT DIAGONALIZATION VIA THE EXTENDED AC-DC ALGORITHM

The AC-DC ("Alternating Columns / Diagonal Centers") algorithm [8] is an alternating directions minimization algorithm, originally intended for the case of a fixed diagonalizing matrix $\boldsymbol{B}$. It alternates between minimization with respect to (w.r.t.) $\boldsymbol{\Gamma}$ and minimization w.r.t. each column of $\boldsymbol{B}$ separately. Having a closed-form solution for a unique global minimizer in each phase, it is guaranteed to converge (under some mild assumptions) to at least a local minimum of the LS criterion.

While in our case the matrix $\boldsymbol{B}$ is not constant, it can be factored as in (6), so as to depend on two constant matrices, $\boldsymbol{A}$ and $\boldsymbol{T}$. As we shall show immediately, it is possible to minimize w.r.t. each column of $\boldsymbol{A}$ and $\boldsymbol{T}$ separately, thus adding another stage to the iterative process.

The extended AC-DC algorithm therefore alternates between minimizations with respect to:

- $\boldsymbol{\Gamma}$ (in the DC phase);
- each column of $\boldsymbol{A}$ (in the AC-1 phase);
- each column of $\boldsymbol{T}$ (in the AC-2 phase).

In each phase, the parameters that do not participate in the minimization are retained fixed at their last value. Some intelligent initial guess should be used as a starting point for all parameters.

3.1. The "DC" phase

In the DC phase we wish to minimize C_{LS} w.r.t. $\boldsymbol{\Gamma}$, with $\boldsymbol{A}$ and $\boldsymbol{T}$ fixed. Since the k-th column of $\boldsymbol{\Gamma}$ is the diagonal of $\boldsymbol{S}_s(\omega_k)$, it participates only in the k-th term of the sum in (10). Thus, in this case the minimization can be decomposed into $K+1$ distinct minimization problems; Moreover, each of these minimization problems is linear in the unknown parameters, and thus admits the well-known linear LS solution. Specifically, note that each (k-th) term in the sum can be expressed as

$$\begin{aligned} &||\boldsymbol{S}_x(\omega_k) - \boldsymbol{B}(\omega_k)\boldsymbol{S}_s(\omega_k)\boldsymbol{B}^H(\omega_k)||_F^2 \\ &\qquad = [\boldsymbol{y}_k - \boldsymbol{H}_k\boldsymbol{\gamma}_k]^H[\boldsymbol{y}_k - \boldsymbol{H}_k\boldsymbol{\gamma}_k] \end{aligned} \tag{12}$$

where $\boldsymbol{\gamma}_k$ is the k-th column of $\boldsymbol{\Gamma}$, $\boldsymbol{y}_k \stackrel{\triangle}{=} \mathrm{vec}\{\boldsymbol{S}_x(\omega_k)\}$ ($\mathrm{vec}\{\cdot\}$ denoting the concatenation of the matrix' columns into one vector), and

$$\boldsymbol{H}_k = (\boldsymbol{B}(\omega_k)^* \otimes \mathbf{1}) \odot (\mathbf{1} \otimes \boldsymbol{B}(\omega_k)) \tag{13}$$

where $\mathbf{1}$ denotes an $L \times 1$ vector of 1-s, $\otimes$ denotes Kronecker's product, $\odot$ denotes Hadamard's (elementwise) product, and the superscript * denotes conjugation (note that this expression is sometimes referred to as the Khatri-Rao product of $\boldsymbol{B}^*$ and $\boldsymbol{B}$). The well-known minimizer of the linear LS problem is

$$\boldsymbol{\gamma}_k = [\boldsymbol{H}_k^H\boldsymbol{H}_k]^{-1}\boldsymbol{H}_k^H\boldsymbol{y}_k. \tag{14}$$

3.2. The "AC-1" phase

We now wish to minimize C_{LS} w.r.t. the l-th ($l = 1, 2, \dots L$) column of $\boldsymbol{A}$, assuming the other columns, as well as $\boldsymbol{T}$ and $\boldsymbol{\Gamma}$, are fixed. Defining

$$\tilde{\boldsymbol{S}}(\omega_k) \triangleq \boldsymbol{S}_x(\omega_k) - \sum_{\substack{n=1\\ n\neq l}}^{L} S_n^s(\omega_k)\boldsymbol{b}_n(\omega_k)\boldsymbol{b}_n^H(\omega_k), \quad (15)$$

where $\boldsymbol{b}_n(\omega_k)$ is the n-th column of $\boldsymbol{B}(\omega_k)$, we have (using the fact that all $S_n^s(\omega_k)$ are real-valued, being the sources' spectrum)

$$\begin{aligned}
C_{LS} &= \sum_{k=0}^{K} ||\tilde{\boldsymbol{S}}(\omega_k) - S_l^s(\omega_k)\boldsymbol{b}_l(\omega_k)\boldsymbol{b}_l^H(\omega_k)||_F^2 \\
&= \sum_{k=0}^{K} Tr\left\{\left[\tilde{\boldsymbol{S}}(\omega_k) - S_l^s(\omega_k)\boldsymbol{b}_l(\omega_k)\boldsymbol{b}_l^H(\omega_k)\right]^H \cdot \right. \\
&\qquad \left. \cdot \left[\tilde{\boldsymbol{S}}(\omega_k) - S_l^s(\omega_k)\boldsymbol{b}_l(\omega_k)\boldsymbol{b}_l^H(\omega_k)\right]\right\} \\
&= \tilde{C} - Tr\left\{\sum_{k=0}^{K} S_l^s(\omega_k)\left[\tilde{\boldsymbol{S}}^H(\omega_k)\boldsymbol{b}_l(\omega_k)\boldsymbol{b}_l^H(\omega_k) + \right.\right. \\
&\qquad \left.\left. + \boldsymbol{b}_l(\omega_k)\boldsymbol{b}_l^H(\omega_k)\tilde{\boldsymbol{S}}(\omega_k)\right]\right\} + \\
&\quad + Tr\left\{\sum_{k=0}^{K} S_l^{s2}(\omega_k)\boldsymbol{b}_l(\omega_k)\boldsymbol{b}_l^H(\omega_k)\boldsymbol{b}_l(\omega_k)\boldsymbol{b}_l^H(\omega_k)\right\} \\
&= \tilde{C} - 2\sum_{k=0}^{K} S_l^s(\omega_k)\boldsymbol{b}_l^H(\omega_k)\tilde{\boldsymbol{S}}(\omega_k)\boldsymbol{b}_l(\omega_k) + \\
&\qquad + \sum_{k=0}^{K}\left(\boldsymbol{b}_l^H(\omega_k)\boldsymbol{b}_l(\omega_k)\right)^2 S_l^{s2}(\omega_k)
\end{aligned} \quad (16)$$

where $\tilde{C}$ is an independent constant. Observe now (from (6)), that $\boldsymbol{b}_l(\omega_k)$ can be written as

$$\boldsymbol{b}_l(\omega_k) = \boldsymbol{\Lambda}_l(\omega_k)\boldsymbol{a}_l \quad (17)$$

where $\boldsymbol{\Lambda}(\omega_k) = \text{diag}\{e^{-j\omega_k\tau_{1l}}, e^{-j\omega_k\tau_{2l}}, \dots e^{-j\omega_k\tau_{Ll}}\}$. Consequently, C_{LS} can be further simplified,

$$C_{LS} = \tilde{C} - 2\boldsymbol{a}_l^T\left[\sum_{k=0}^{K} S_l^s(\omega_k)\boldsymbol{\Lambda}_l^H(\omega_k)\tilde{\boldsymbol{S}}(\omega_k)\boldsymbol{\Lambda}_l(\omega_k)\right]\boldsymbol{a}_l + (\boldsymbol{a}_l^T\boldsymbol{a}_l)^2\sum_{k=0}^{K} S_l^{s2}(\omega_k). \quad (18)$$

We can further decompose $\boldsymbol{a}_l$ into a scale a times a unit-norm vector $\boldsymbol{\alpha}$ ($\boldsymbol{a}_l = a\boldsymbol{\alpha}$, such that $\boldsymbol{\alpha}^T\boldsymbol{\alpha} = 1$), thus reducing (18) into

$$C_{LS} = \tilde{C} - 2a^2\boldsymbol{\alpha}^T\boldsymbol{F}\boldsymbol{\alpha} + a^4 f \quad (19)$$

where $\boldsymbol{F}$ is the Hermitian matrix

$$\boldsymbol{F} \triangleq \sum_{k=0}^{K} S_l^s(\omega_k)\boldsymbol{\Lambda}_l^H(\omega_k)\tilde{\boldsymbol{S}}(\omega_k)\boldsymbol{\Lambda}_l(\omega_k) \quad (20)$$

and

$$f = \sum_{k=0}^{K} S_l^{s2}(\omega_k). \quad (21)$$

Differentiating (18) w.r.t. a and equating zero yields either the solution $a = 0$ or

$$a^2 = \boldsymbol{\alpha}^T\boldsymbol{F}\boldsymbol{\alpha}/f. \quad (22)$$

Since $\boldsymbol{F}$ is Hermitian, (22) is real-valued. Thus, if (22) is positive, then the minimizing a is the square root, otherwise it is zero. Consequently, if $\boldsymbol{F}$ is negative-definite, then minimization of C_{LS} w.r.t. $\boldsymbol{a}_l$ is attained by $\boldsymbol{a}_l = \boldsymbol{0}$. Normally, however, this is not the case, and substituting a^2 back into (19) reduces the problem into minimization w.r.t $\boldsymbol{\alpha}$ of

$$C_{LS} = \tilde{C} - (\boldsymbol{\alpha}^T\boldsymbol{F}\boldsymbol{\alpha})^2/f \quad (23)$$

subject to $\boldsymbol{\alpha}^T\boldsymbol{\alpha} = 1$. The desired solution is attained as the eigenvector of $\boldsymbol{F}$ associated with the largest (*positive*) eigenvalue.

3.3. The "AC-2" phase

It is now desired to minimize C_{LS} w.r.t. $\boldsymbol{\tau}_l$, the l-th ($l = 1, 2, \dots L$) column of $\boldsymbol{T}$, assuming the other columns, as well as $\boldsymbol{A}$ and $\boldsymbol{\Gamma}$ are fixed. Since the dependence on the delays $\boldsymbol{\tau}_l$ in only through $\boldsymbol{\Lambda}_l(\omega_k)$, it is evident from (18) that C_{LS} can be expressed as

$$C_{LS} = \tilde{\tilde{C}} - 2\boldsymbol{a}_l\left[\sum_{k=0}^{K}\boldsymbol{\Lambda}_l^H(\omega_k)\boldsymbol{G}(\omega_k)\boldsymbol{\Lambda}_l(\omega_k)\right]\boldsymbol{a}_l \quad (24)$$

where $\tilde{\tilde{C}}$ is another independent constant, and the matrix $\boldsymbol{G}(\omega_k)$ is defined as

$$\boldsymbol{G}(\omega_k) \triangleq S_l^s\tilde{\boldsymbol{S}}(\omega_k). \quad (25)$$

Differentiating w.r.t. τ_{pl} (for $p = 1, 2, \dots L$ except for $p = l$) and equating zero, we obtain the following set of equations:

$$\begin{aligned}
\frac{\partial C_{LS}}{\partial \tau_{pl}} &= -2j\sum_{m=1}^{L} a_{pl}a_{ml} \cdot \\
&\cdot \sum_{k=0}^{K}\omega_k\left(g_{pm}(\omega_k)e^{j\omega_k(\tau_{ml}-\tau_{pl})} - \right. \\
&\left. - g_{mp}(\omega_k)e^{-j\omega_k(\tau_{ml}-\tau_{pl})}\right) = 0 \\
&\qquad 1 \le p \le L, p \neq l
\end{aligned} \quad (26)$$

where a_{ij} and $g_{ij}(\omega_k)$ denote the i,j-th elements of $\boldsymbol{A}$ and $\boldsymbol{G}(\omega_k)$, respectively. This set of equations is to be solved w.r.t. $\tau_{1l}, \tau_{2l}, \ldots \tau_{Ll}$ except for τ_{ll}, which by convention was set to zero.

We do not have an analytical solution to (26) for the general case. However, if we reduce the discussion to the case of $L = 2$ sensors and sources, and the frequencies $\{\omega_k\}_{k=0}^{K}$ are chosen as

$$\omega_k = k\Omega \quad k = 0, 1, \ldots K, \tag{27}$$

(with Ω a selected constant), then this set reduces to:

$$\sum_{k=0}^{K} k\left[g_{21}(k\Omega)e^{-j\Omega\tau_{21}k} - g_{12}(k\Omega)e^{j\Omega\tau_{21}k}\right] = 0 \tag{28a}$$

(for $l = 1$, $p = 2$), and

$$\sum_{k=0}^{K} k\left[g_{12}(k\Omega)e^{-j\Omega\tau_{12}k} - g_{21}(k\Omega)e^{j\Omega\tau_{12}k}\right] = 0 \tag{28b}$$

(for $l = 2$, $p = 1$). To proceed, we now define $\rho_{pl} \triangleq e^{j\Omega\tau_{pl}}$, so that (28a,28b) can be written as

$$\sum_{k=0}^{K} k[g_{pl}(k\Omega)\rho_{pl}^{-k} - g_{lp}(k\Omega)\rho_{pl}^{k}] = 0, \tag{29}$$

which, after multiplication by ρ^K turns into a polynomial of degree $2K$ in ρ. Using polynomial rooting and selecting all unit-modulus roots $\hat{\rho}_{pl}$, yields all (possibly numerous) stationary points of $C_L S$ w.r.t. τ_{pl} via

$$\hat{\tau}_{pl} = \mathrm{Imag}\{\log \hat{\rho}_{pl}\}/\Omega. \tag{30}$$

Each of these candidate solutions can be plugged into (24) for evaluation of C_{LS} in order to select the minimizing solution.

It is interesting to observe, that in the $L = 2$ case the dependence of (26) on $\boldsymbol{A}$ vanishes, so that this phase (AC-2) can be regarded an inseparable part of the previous phase (AC-1), since minimization w.r.t. both $\boldsymbol{a}_l$ and $\boldsymbol{\tau}_l$ can be attained simultaneously.

4. RECONSTRUCTION OF THE SOURCE SIGNALS

While estimates of the mixing parameters (especially of the delays) may be of prime interest in certain applications, it may often be desired to actually separate the source signals. Using the estimated mixing parameters, this can be conveniently done in the frequency domain.

Using the Discrete Fourier Transform (DFT) of the observations, we obtain L length-N series,

$$y_p[m] = \sum_{n=1}^{N} x_p[n]e^{-j2\pi(n-1)m/N}$$
$$p = 1, 2, \ldots L \quad m = 0, 1, \ldots N-1, \tag{31}$$

Denoting by $\hat{\boldsymbol{A}}$ and $\hat{\tau}_{kl}$ the estimated mixing coefficients and delays, respectively, we construct matrices $\hat{\boldsymbol{D}}[m]$, whose k,l-th element is given by

$$\hat{D}_{kl}[m] = \begin{cases} e^{-j2\pi m\hat{\tau}_{kl}/N} & 0 \le m \le N/2 \\ e^{-j2\pi(m-N)\hat{\tau}_{kl}/N} & N/2 < m \le N-1 \end{cases} \tag{32}$$

The sources can now be separated in the frequency domain using

$$\boldsymbol{z}[m] = \left[\hat{\boldsymbol{A}} \odot \hat{\boldsymbol{D}}[m]\right]^{-1} \boldsymbol{y}[m] \tag{33}$$

where $\boldsymbol{y}[m] \triangleq [y_1[m]y_2[m]\cdots y_L[m]]^T$ (assuming the inverse exists). Transferring back to the time-domain,

$$\hat{s}_p[n] = \frac{1}{N}\sum_{m=0}^{N-1} z_p[m]e^{j2\pi m(n-1)/N} \quad n = 1, 2, \ldots N \tag{34}$$

where $z_p[m]$ is the p-th element of $\boldsymbol{z}[m]$. Apart from end-effects near $n = 1$ and $n = N$, $\hat{s}_p[n]$ are reconstructed versions of the sources $s_p[n]$, possibly with arbitrary permutation, scaling and delay, due to the inherent indeterminancies of the blind scenario.

5. SIMULATIONS RESULTS

In this section we present simulations results for two experiments. In the first experiment we compare the accuracy in estimating the delays to that of another second-order statistics method proposed by Comon and Emile in [5]. In the second experiment we present our algorithm's performance in terms of the attained signal separation (the residual signal to interference ratio (SIR)).

For both experiments we used source signals generated as follows. Both signal were originally generated at a sample rate 10 times higher then the eventual processing sample rate, to enable fractional delays prior to "sampling". At the higher sample rate the signals were first generated as first-order Auto-Regressive (AR(1)) processes with parameters 0.76 and 0.81 for the first and second signals, respectively. The driving noise was zero-mean unit variance white Gaussian noise. Then, to enable subsequent decimation by 10 without aliasing, both signals were low-pass filtered to a maximum frequency of $0.7 \cdot 2\pi/10$ using a Kaiser-windowed filter of 80 taps with window parameter $\beta = 3.44$. This provides for stop-band attenuation of about 40dB with a transition band of about $\pi/70$ (see, e.g. [10] for Kaiser windows). The signals were then delayed and mixed, prior to subsequent decimation by 10.

For the first experiment we used as a mixing matrix $\boldsymbol{A} = \left[\begin{smallmatrix} 1 & 1 \\ 1 & 1 \end{smallmatrix}\right]$, to enable comparison to the algorithm of [5], which can only accommodate this mixing matrix. Note also, that [5] implicitly uses the knowledge of this mixing matrix, whereas our algorithm attempted to estimate these mixing parameters as well. The results in terms of the mean squared error (mse) in estimating the delays vs. the sample length N are shown in figure 1. Both algorithms used the same data, with 100 trials for each sample length. The "pulsation parameter" for the algorithm of [5] was $\omega = 0.245$. The true nonzero delays were $\tau_{12} = 2.1, \tau_{21} = 5.5$.

For the second experiment we chose a non-singular mixing matrix, $\boldsymbol{A} = \left[\begin{smallmatrix} 1 & 0.44 \\ -0.71 & 1 \end{smallmatrix}\right]$, which, unlike the singular mixing matrix of the first experiment, enables demixing at all frequencies (a singular matrix disables demixing at least at frequency 0). The true nonzero delays were $\tau_{12} = 4.2, \tau_{21} = -6.7$. In figure (2) we present the residual SIR (averaged over 100 trials) vs. the sample length. It is to be noted, that the accurate estimation of delays significantly improves the attainable separation. For example, the performance in figure 2 may be compared to the separation attained when the true mixture parameters are known, and the true delays are only known up to their integer part (which can be roughly deduces from cross-correlating the signals). This SIR was measured empirically to be 34.5dB for the first source and 23dB for the second (considerably lower than our algorithm's performance in figure 2.

Our algorithm was used (in both experiments) with $K = 10$ matrices at 10 frequencies with $\Omega = 0.25$ spacing. We used the identity matrix as an initial guess for the mixing coefficients, and the integer values of the true delays as initial guesses for the delays. The initial guesses for the sources spectra were obtained as a result of the first DC phase. Convergence was usually attained within up to 15 iterations, depending on the sample length N

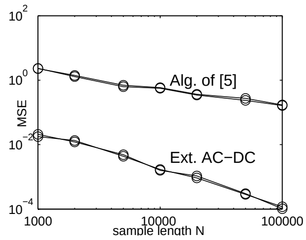

Fig. 1: delays estimation MSE

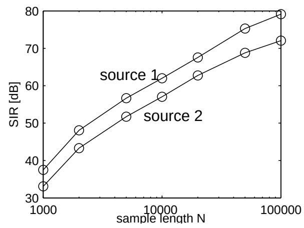

Fig. 2: Residual SIR

6. CONCLUSION

We presented an algorithm for blind separation of pure-delay mixtures with unknown delays and mixture coefficients. The algorithm is based on approximate joint diagonalization of matrices of estimated auto- and cross-spectra. It uses an iterative alternating-coordinates scheme to minimize w.r.t. the unknown source spectra (termed "DC" phase), the unknown mixing coefficients ("AC-1" phase) and the unknown delays ("AC-2" phase). We derived closed-form minimization for the DC and AC-1 phases for the general case, but for the "AC-2" phase we only provided a solution for the two sensors / two sources case.

The algorithm's performance was shown to offer considerable improvement, at least with respect to less exhaustive methods, such as observing delays from the cross-correlations, or using the second-order statistics algorithm of [5]. However, many aspects of the attainable performance (for example, in terms of possibly optimal/adaptive frequency selection and proper weighting of the LS criterion) are still under study.

7. REFERENCES

[1] Jean-François Cardoso, "Blind signal separation: statistical principles," *Proceedings of the IEEE*, vol. 86, no. 10, pp. 2009–2025, 1998.

[2] D. Yellin and E. Weinstein, "Multichannel signal separation: methods and analysis," *IEEE Trans. Signal Processing*, vol. 44, no. 1, pp. 106–118, 1996.

[3] K. I. Diamantaras, A. P. Petropulu, and B. Chen, "Blind two-input-two-output FIR channel identification based on frequency domain second-order statistics," *IEEE Trans. Signal Processing*, vol. 48, no. 2, pp. 534–542, 2000.

[4] Kamran Rahbar and James P. Reilly, "Blind source separation of convolved sources by joint approximate diagonalization of cross-spectral density matrices," *Proceedings ICASSP 2001*, vol. 5, 2001, Salt Lake City.

[5] P. Comon and B. Emile, "Estimation of time delays in the blind mixture problem," *Proceedings of EUSIPCO'94*, pp. 482–485, 1994, Edinbourgh, Scotland.

[6] B. Emile, P. Comon, and J. Le Roux, "Estimation of time delays with fewer sensors than sources," *IEEE Trans. Signal Processing*, vol. 46, no. 7, pp. 2012–2015, 1998.

[7] B. Emile and P. Comon, "Estimation of time delays between unknown colored signals," *Signal Processing*, vol. 69, pp. 93–100, 1998.

[8] Arie Yeredor, "Approximate joint diagonalization using non-orthogonal matrices," *Proceedings ICA2000*, pp. 33–38, 2000.

[9] S. M. Kay, *Fundamentals of Statistical Signal Processing*, Prentice-Hall, 1993.

[10] B. Porat, *A Course in Digital Signal Processing*, John Wiley & Sons, 1997.